Invariance of momentum and energy conservation laws under Galilean transformation

By

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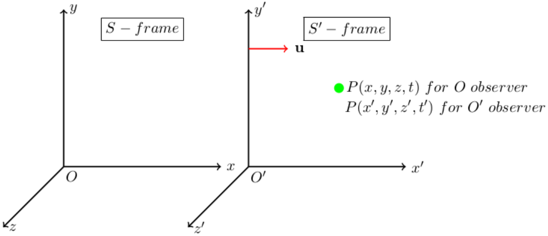
Outline

Galilean transformation

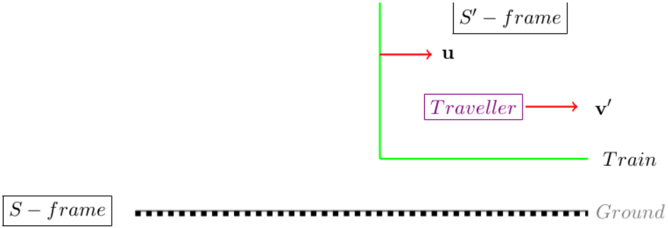
An event is specified by point P in the figure below.



$$x' = x - ut, \quad y' = y, \quad z' = z; \quad t' = t \; .$$

Vectorial form : $\mathbf{x}' = \mathbf{x} - \mathbf{u}t; \quad t' = t.$

Velocity addition theorem



$$v'_x = v_x - u, \quad v'_y = v_y, \quad v'_z = v_z,$$

Vectorial form : $\mathbf{v}' = \mathbf{v} - \mathbf{u}.$

Momentum Conservation

Total momentum before collission(P_{bc})=Total momentum after collission (P_{ac}) -in all Inertial frame

Let us consider head-on collission of two objects along x-axis of S frame (I have taken here $1d$ case you can generalized it to $3d$. Use vectorial form of transformation rules)

For S frame observer:

$$P_{bc} = P_{ac} \tag{1}$$

$$\boxed{m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2}$$

For S' farme observer:

The corresponding quantities are denoted by prime on it. Note that mass is invariant for the both observers and we do not put prime on m_1 and m_2.

Now apply velocity transformation rule $v_1' = v_1 - u$, $V_1' = V_1 - u$, etc.

$$m_1(v_1' + u) + m_2(v_2' + u) = m_1(V_1' + u) + m_2(V_2' + u)$$

$$\boxed{m_1 v_1' + m_2 v_2' = m_1 V_1' + m_2 V_2'}$$

Energy Conservation

Consider same example-elastic collission of two object.

For S-frame observer:

$$\boxed{\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2}$$

For S' frame observer use velocity transformation rule

$$\frac{1}{2}m_1(v_1' + u)^2 + \frac{1}{2}m_2(v_2' + u)^2 = \frac{1}{2}m_1(V_1' + u)^2 + \frac{1}{2}m_2(V_2' + u)^2$$

Or we can write

$$\frac{1}{2}m_1(v'^2_1 + u^2 + 2v'_1u) + \frac{1}{2}m_2(v'^2_2 + u^2 + 2v'_2u)$$
$$= \frac{1}{2}m_1(V'^2_1 + u^2 + 2V'_1u) + \frac{1}{2}m_2(V'^2_2 + u^2 + 2V'_2u)$$

$$\frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 + (m_1v'_1 + m_2v'_2)$$
$$= \frac{1}{2}m_1V'^2_1 + \frac{1}{2}m_1V'^2_2 + (m_1V'_1 + m_2V'_2)$$

or

$$\boxed{\frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 = \frac{1}{2}m_1V'^2_1 + \frac{1}{2}m_1V'^2_2}$$

Thank You

Stay Home Stay Safe