Quantum Mechanics-Section5

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1 Particle nature of waves

1.1 Photoelectric Effect: Quantization of Light

1.1.1 Failure of classical theory

The well-established classical electrodynamics (based upon Maxwell's equations) has given the idea that electromagnetic wave is composed of sinusoidal electric filed vector $\overrightarrow{E}$ and magnetic field $\overrightarrow{B}$ where both lie on mutually perpendicular planes that contain the propagation vector $\overrightarrow{k}$. The energy density u associated to the electromagnetic wave is :

$$u(E,B) = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

In other words, the energy of an electromagnetic wave depends on its intensity (I), at least classical electrodynamics tell us that. According to this theory, photoemission must take place immediately, if the incident radiation has sufficient intensity. Even if the intensity is low, photoemission must occur, if the irradiation is projected on the emitting plate for long enough duration. In contradiction to this classical theory, it was experimentally observed that even a faint beam (low intensity) of light with frequency higher than ν_{th} can eject photoelectrons, while a highly intense beam with frequency lower than ν_{th} can not. Therefore, the classical wave model of light is unable to explain why photoemission process is governed by the frequency ν_{th} of light and not the intensity (I). Moreover, many other questions, *e.g.* why the stopping potential V_S (i.e. the maximum kinetic energy of the photoelectron $\frac{1}{2}mv_{max}^2$=eV_S) depends on frequency ν of the incident light? Why ν_{th} is material dependent? Why the proportional change of the stopping potential with frequency is material independent? - remains unanswered by the classical theory.

1.1.2 Einstein's explanation

This failure of the classical mechanics was resolved by the quantum theory of light invented by Albert Einstein in 1905. Einstein proposed that light is composed of 'energy packets' or 'quanta'. The quanta of light is called 'Photon'. A light of frequency ν is made up of such energy packets or photons where each photon has energy $h\nu$. In brief, quantum theory provides the discrete or particle nature of light. Let's discuss how Einstein explained the crucial observations related to photoemission. In simple language, photoemission is 'photon in, electron out' phenomena as shown in figure (1). Within the atom, the electrons are bound to the nucleus while they revolve around - the binding energy is E_B. When a photon of energy hν is absorbed, a part of that energy is expended to remove the electron from atomic bond and a larger portion of the remaining energy becomes the kinetic energy of the emitting electron $\frac{1}{2}mv^2$. Therefore, it is obvious that, if the binding energy of the electron is less then its kinetic energy will be higher. In case, if the electron binding energy is zero, then its kinetic energy is maximum. Inside a metallic crystal, there are many free electrons (electrons with zero binding energy) at finite temperature. But, these free electrons are bound to the crystal and therefore can not come out of the crystal spontaneously. This binding of the electron to the crystal is expressed in terms of *work function* ϕ of the material. Therefore, the photon energy $h\nu$ may be expressed as:

$$h\nu = E_B + \frac{1}{2}mv^2 + \phi$$

For free electrons inside metal E_B=0, therefore, the kinetic energy becomes maximum, and we may write:

$$h\nu = \frac{1}{2}mv_{max}^2 + \phi$$

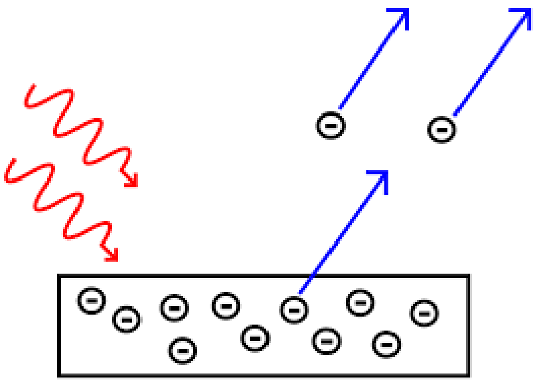

Figure 1: Photoemission: photon (shown in the form of wave) in, electron out

this correlation of the photon energy to the maximum kinetic energy of the photoelectrons was drawn by Einstein, where ϕ is the work function of the plate/cathode material. The above equation may also be written as,

$$h\nu - \phi = \frac{1}{2}mv_{max}^2$$

. If $\phi = h\nu_{th}$, then

$$h(\nu - \nu_{th}) = \frac{1}{2}mv_{max}^2$$

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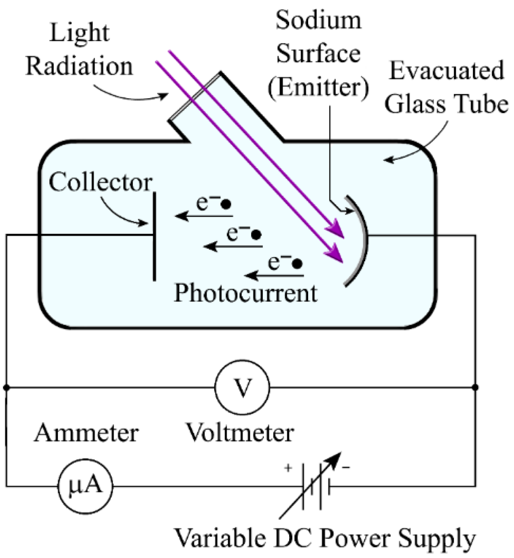

Figure 2: A circuit diagram for Photoelectric effect experiment

(A) The stopping potential V_S *i.e.* the maximum kinetic energy of the electrons $\frac{1}{2}mv_{max}^2$ (=eV_S) depends on frequency of the incident photon ν. Therefore, we measure different stopping potentials V_S^1, V_S^2 and V_S^3 while incident light frequencies are ν_1, ν_2 and ν_3 respectively. This we have seen earlier (read previous sections).

(B) Einstein's equation of photoelectric effect clearly shows that, for photoemission to occur (i.e. $\frac{1}{2}mv_{max}^2$>0) the condition is $h\nu > h\nu_{th}$, i.e. the incident photon frequency has to be greater than the threshold frequency. This equation also explains, how the stopping potential V_S (or, $\frac{1}{2}mv_{max}^2$) depends linearly on the incident photon frequency. The maximum kinetic energy of the photoelectrons decreases as the incident photon frequency ν decreases.

When ν decreases down to ν_{th} then ϕ=hν_{th}, therefore $\frac{1}{2}mv_{max}^2$=0. So the stopping potential also becomes zero. As a result, no photoelectrons can come out of the metal surface, as we have seen in earlier graphs.

(C) If the incident radiation frequency is just higher than the threshold frequency ν_{th} photoemission process starts. Under this condition, if the intensity of irradiation increases then the incident photon density becomes higher. As a result, the number of emitted photoelectrons increases which results in higher circuit current.

(D) In another graph shown in previously, we observe that the slope of the linear variation of stopping potential with frequency is same for all metal. From Einstein's photoelectric equation we may easily derive that $\frac{\partial V_S}{\partial \nu}$ is constant. Therefore, as per expectation, the slope should not depend on material properties.

Einstein's quantum model of light was successful to explain all the aspects of photoelectric effect. In fact, this model opened up a whole new branch of physics later, called *Quantum Optics*. Notably, in the definition that says-light is made up of small energy *packets*, Einstein cautiously avoided the term 'particles' - just to distinguish it from our well-known classical particles. Photon has zero rest mass.

References

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[1] Figures are collected from online resources.