Quantum Mechanics-Section9

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1 Wave nature of particles

Similar to the fact that radiation wave demonstrates particle-like nature (as we have seen for the cases of Phtoelectric effect and Compton effect), quantum mechanical particles also demonstrate wave-like behaviour.

1.1 Matter waves: de-Broglie's hypothesis

In 1924, Louis de Broglie, a French theoretical physicist proposed that just like the radiations, matters also show wave-particle dual nature. Like a photon has an associated wave that governs its motion, matter (*e.g.* an electron) also has an associated matter-wave with it. In fact, the universe is made of matter and energy, de-Broglie's theory speaks of the grand symmetry of universe. He proposed that wave aspects of matter is similar, in the same quantitative way, to the particle aspects of radiation.

According to de-Broglie's hypothesis, the energy E of any physical quantity (both matter and radiations) is related to the frequency ν of the wave associated with its motion in the following way:

$$E = h\nu$$

This equation may also be written (since, ω=$2\pi\nu$) in different form:

$$E = \hbar\omega$$

The momentum p of the entity is related to the wavelength λ of the corresponding wave in the following way:

$$p = \frac{h}{\lambda}$$

Here λ is the *de-Broglie wavelength*. This equation may also be written (since, k=$\frac{2\pi}{\lambda}$) in other form like:

$$p = \hbar k$$

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The wave nature of light remains hidden during the experiments of geometrical/ray optics, since the optical apparatus (slit width, lens width *etc.*) used in such cases have dimensions much greater than the de-Broglie wavelength λ. Lets say 'a' represents the slit width of an optical diffraction experiment, whereas λ is the wavelength of the used light. The angle of diffraction θ depends on the factor $\frac{\lambda}{a}$. If the slit width a$>>\lambda$, that happens in case of ray optics, $\frac{\lambda}{a} \to 0$, the diffraction angle is so small that diffraction effect is negligible. Therefore, geometrical optics speaks of ray propagation, similar to trajectory motion of classical particles.

On the other hand, when the aperture or slit width 'a' is so narrow that its dimension is comparable to or even smaller than the wavelength λ of the light that passes through it,$\frac{\lambda}{a} \approx 1$, the situation is under the domain of wave optics. The diffracting angle $\theta = \frac{\lambda}{a}$ is large enough for making the diffraction effects apparent. In case of matter, its wave nature will be dominant when it passes through apertures or interacts with obstacles of dimension comparable to the dimension of the corresponding de-Broglie wavelength λ. As we know that the de-Broglie wavelength $\lambda = \frac{h}{p}$, the lower momentum of the particle corresponds to larger λ triggering the wave aspects of matter to be dominant. For objects with high momentum (p), the corresponding de-Broglie wavelength is too small to demonstrate wave properties. In case of an electron, its tiny mass results into an appreciably lower momenta p, therefore the corresponding λ is of the order of 1 Å. In such a case, matter waves of electron diffract when electron passes through a crystal, where the inter-planner distance is of angstrom order, therefore $\lambda \approx a$. On the other hand, p is larger for a tennis ball, this results into a corresponding λ of the order of 10^{-25} Å. Diffracting apparatus of such a low dimension is not available, therefore, tennis ball is unexpected to demonstrate wave properties.

References

[1] Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles, Authors: R. Eisberg, R. Resnick, Publisher: Wiley

[2] Quantum Physics, Author: Stephen Gasiorowicz, Publisher: John Wiley and Sons

[3] Quantum Mechanics: Theory and Applications, Authors: A. Ghatak, S. Lokanathan, Publisher: Trinity Press

[4] Modern Quantum Mechanics, Author: J.J. Sakurai, Publisher: Benjamin/Cummings Publisher.

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[1] Figures are collected from online resources.