Quantum Mechanics-Section10

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0.1 Davission-Germer experiment

With an idea that - the wave nature of matter may be tested in a same way the wave nature of x-ray was first tested, Davission and Germer performed electron diffraction experiments. Electrons of sufficient energy was allowed to fall on single crystal, where the crystal behaves as a three dimensional array of scattering centres. The electron matter-waves are scattered strongly to few characteristic directions just as x-ray.

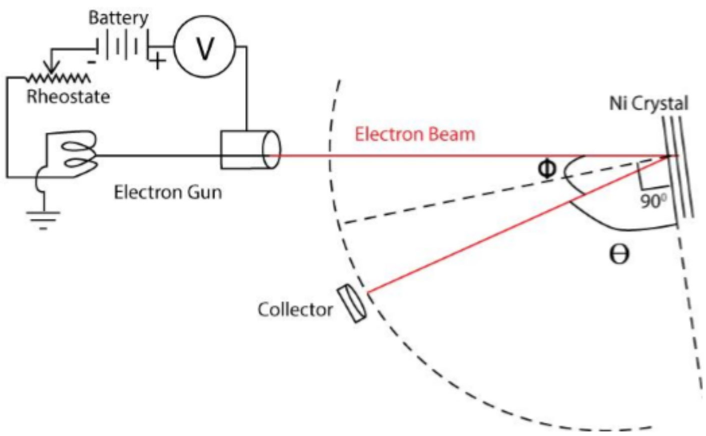

Figure 1: The arrangement for electron scattering in Davission-Germer experiment

In figure (1), the experimental arrangement for electron diffraction by Davission and Germer is shown. The schematic diagram shows that electrons are originated by thermal emission from the heated filament and they are accelerated by a potential V. These electrons therefore move forward with a kinetic energy eV and fall on a Nickel single crystal. After getting scattered by the single crystal these electrons are detected by a movable electron detector at a scattering angle ϕ. Davission and Germer performed the intensity measurement of the scattered electrons for various scattering angles ϕ, while the kinetic energies of the incident electron was also varied by changing the accelerating potential V.

The results of their rigorous experiment is shown in figure (2), where a maximum intensity of the scattered electrons is observed at an angle θ=50^0 for the value of accelerating potential V=54 Volt.

The existence of peak intensity at a particular angle indicates to the constructive interference of the scattered waves from different atoms periodically arranged inside the crystal - which is basically electron diffraction. Diffraction is a property demonstrated by waves only, classical particles do not take part in diffraction process. We have seen that x-ray scattering from single crystals create diffraction patterns obeying the Bragg's law. In this present case, the matter-waves or de-Broglie waves corresponding to the electrons of a particular energy/momentum produces diffraction pattern. Inside the Ni crystal, there are parallel atomic planes where the atoms are arranged in a regular periodic fashion, they work as scatterers. The matter waves are scattered from various set of parallel planes and produce diffraction pattern. It is not that the diffraction occurs due to scattering of matter-waves corresponding to different electrons, but different parts of the matter wave corresponding to a single electron causes the diffraction. This may be realized if the density of incoming electrons is lowered so that only one electron comes out of the electron

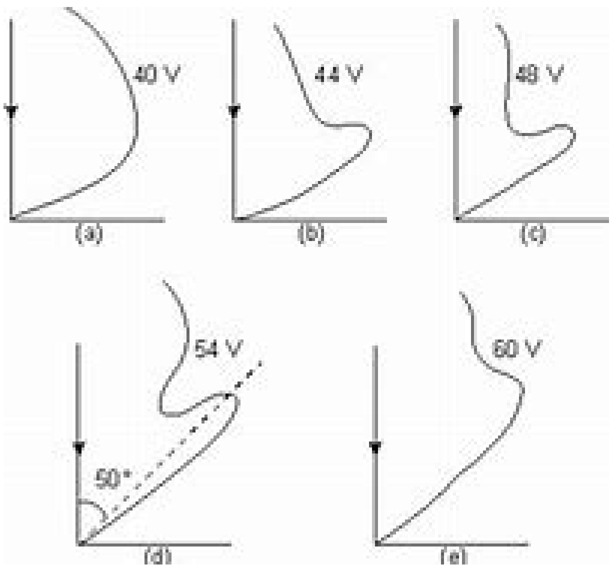

Figure 2: Scattered electron intensity plot for different scattering angles and different electron kinetic energies. Maximum intensity: θ=50^0, V=54 V.

gun at a time and gets scattered from the crystal. This will also create the same diffraction pattern but of weak intensity.

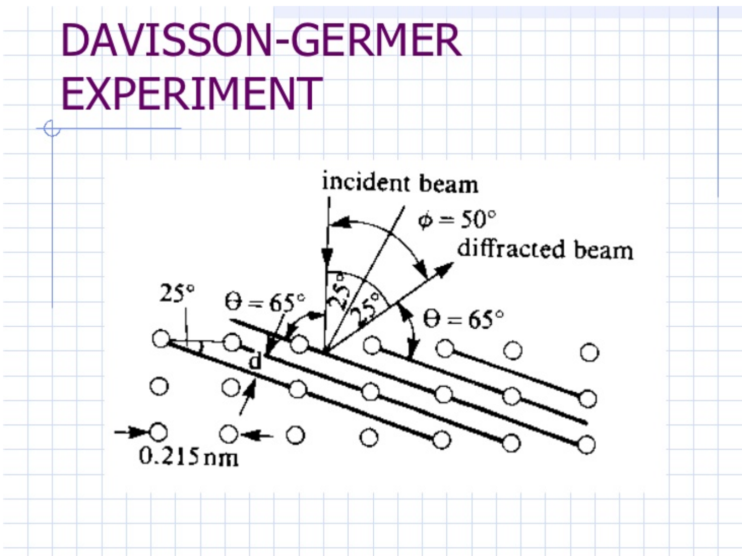

Figure 3: Bragg diffraction of matter waves corresponding to the incident electrons. The electrons diffract from various set of parallel planes with inter-planner spacing d.

We can apply the theory of Bragg diffraction to find out the wavelength of the matter wave of the incident electrons. The Bragg angle as per geometry shown in figure (3) is: θ=(180-50)/2=65^0. According to the Bragg's law:

$$2d sin\,\theta = n\lambda$$

In this equation if we put θ=65^0 and the inter-planner spacing may be obtained from the x-ray diffraction on the same crystal to be 0.91Å. Putting n=1, we may calculate the magnitude of λ.

$$\lambda = 2 \times 0.91 \times sin\,65^0 = 1.65$$

The magnitude of λ may be obtained using de-Broglie's equation.

$$\lambda = \frac{h}{p}$$

As we know,

$$p = \sqrt{2mE}$$

Here E is the kinetic energy of the incident electron

$$E = eV$$

Therefore,

$$p = \sqrt{2meV}$$

Hence, the corresponding de-Broglie wavelength is:

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Putting the value of h=6.626×10^{-34} J.s and V=54 volt, we obtain:

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$$\lambda = 1.65$$

This impressive result $\lambda = 1.65$Å obtained from Bragg's law as well as de-Broglie's matter-wave theory proves the existence of matter-waves.

Not only electrons, but all the material objects demonstrate the wave nature. G P Thomson independently explored the wave property of electron while it passes through thin films and there occurs diffraction. The wave property of hydrogen molecule and helium atom beams were also discovered by Estermann, Stern and Frisch during scattering experiment on Lithium Fluoride crystal. Diffraction phenomena was observed for the case of neutrons, which are quite heavier than electrons, by Fermi, Marshall and Zinn. The existence of matter waves is a well established fact.

Both matter and radiation have wave and particle nature. The particle aspect of a physical entity dominates while they interact (emit, absorbed or collide) with matter, while their wave nature dominates during their propagation through a system. For matter, to display their wave properties its momenta p has to be low enough, so that the corresponding de-Broglie wavelength λ is sufficiently large to be measurable and we are under the domain of physical optics. For macroparticles, their mass is so large that a little velocity contributes towards a high magnitude of p. As a result they produce very short λ, (since $\lambda = \frac{h}{p}$) which is beyond the measurable limit. Their wave property is, therefore, supressed. These macroparticles dominantly demonstrate material/particle nature.

References

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[1] Figures are collected from online resources.