Quantum Mechanics-Section 13

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1 The Uncertainty Principle

We have already seen that both radiations and matters behave like waves as well as particles. The consequence of wave-particle duality leads to the uncertainty principle which states:

If the x-coordinate of the position of a particle is known to an accuracy Δx, then the component of the momentum can not be determined to an accuracy better than $\Delta p_x \approx \frac{\hbar}{\Delta x}$, where $\hbar = \frac{h}{2\pi}$ and h is Planck's constant.

Alternative, we can say, if Δx and Δp_x represent the accuracy with which the x-coordinate of the position and x-component of the momentum can be determined, then the following inequality must be satisfied

$$\Delta x \Delta p_x \geq \hbar$$

While constructing the matter waves we established the relationship

$$\Delta x \Delta k_x \geq 1$$

We may use de-Broglie's hypothesis $p_x = \hbar k_x$, to prove

$$\Delta x \Delta p_x \geq \hbar$$

A wave packet is composed of waves of different wavelengths. The superposition of multiple waves of various wavelengths construct a modulated wave as we can see in figure (1). Higher the number of superposing waves, higher will be exponential decay of amplitude from its peak value in the resulting wavepacket thereby making the matter wave more localized. As we know, that a wavepacket represents the position of a particle probabilistically, its large peak value with sharply decaying amplitude corresponds to higher accuracy in measurement of the corresponding particle's position. Therefore, the particle's position x in the one dimensional case may be determined more accurately while the Δx is lower. On the other hand, the requirement for more waves with various wavelengths makes range of λ higher. Since de-Broglie's principle states $p = \frac{h}{\lambda}$, greater range of λ results to higher range for momentum p. This, in turn, makes Δp higher. It is therefore clear that one can not lower both Δp and Δx simultaneously.

Since the width represents a region in which a particle is likely to be in x-space or in momentum space, uncertainty principle states that if try to construct a highly localized wave packet in x-space, then it is impossible to associate a highly localized momentum with it, contradicting the classical physics. A wave packet characterized by a momentum defined within narrow limits must be spatially very broad. The classical concept of position and momentum are independent of each other. They refer to different degrees of freedom (q_i and p_i). In quantum mechanics, position and momentum are complementary properties of a system and the theory does not admit the possibility of an experiment in which both can be established simultaneously. The smallness of $\hbar$ ensures that only for microscopic systems the usual notion of classical physics will fail. For example, for a dust particle of mass 10^{-4} gm moving with a velocity of 10^4 cm/sec with an uncertainty in the product of one part in a million implies $\Delta p \approx 10^{-6}$ and thus $\Delta x \approx 10^{-21}$ cm, which is 10^8 times smaller than the radius of a proton. But, in case of an electron in Bohr orbit the situation is different. Here, the order of magnitude of Δx is about the radii of the orbit.

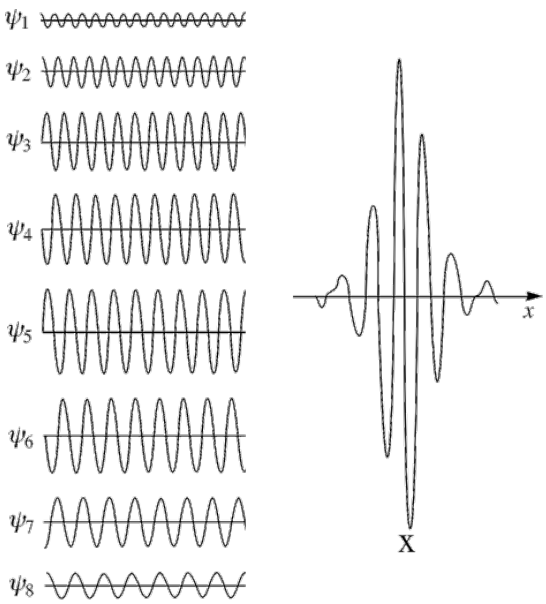

Figure 1: Construction of wavepacket which represents matter wave. A wavepacket is the superposition of several waves with different wavelengths.

References

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[1] Figures are collected from online resources.