# Quantum Mechanics-Section 15

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## 0.1 Construction of Schrodinger Equation using wave-packet

A wave packet may be expressed as:

$$\psi(x,t) = \int dk\, g(k)\, e^{ikx-i\omega t}$$

Using de Broglie's hypothesis $p = \hbar k$ and $E = \hbar\omega$ we may express the above equation in terms of $p$ and $E$:

$$\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p)\, e^{i(px-Et)/\hbar}$$

We have already seen that the group velocity $v_g$ associated to a particle may be expressed as

$$v_g = \frac{d\omega}{dk} = \frac{p}{m}$$

A time derivative of the wavefunction $\psi(x,t)$ gives:

$$\frac{\delta\psi(x,t)}{\delta t} = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p) e^{i(px-Et)/\hbar} \left(\frac{-iE}{\hbar}\right)$$

$$i\hbar\frac{\delta\psi(x,t)}{\delta t} = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p) E e^{i(px-Et)/\hbar}$$

$$i\hbar\frac{\delta\psi(x,t)}{\delta t} = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p) \frac{p^2}{2m} e^{i(px-Et)/\hbar} \quad (1)$$

Differentiating the wave function $\psi(x,t)$ with respect to $x$:

$$\frac{\delta\psi(x,t)}{\delta x} = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p)\, e^{(i(px-Et)/\hbar)} \left(\frac{ip}{\hbar}\right)$$

Differentiating again,

$$\frac{\delta^2\psi(x,t)}{\delta x^2} = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p)\, e^{(i(px-Et)/\hbar)} \left(\frac{ip}{\hbar}\right)^2$$

$$\frac{\delta^2\psi(x,t)}{\delta x^2} = \frac{1}{\sqrt{2\pi\hbar}} \int dp\, \phi(p)\, e^{(i(px-Et)/\hbar)} \left(\frac{-p^2}{\hbar^2}\right) \quad (2)$$

Comparing equation (1) and equation (2) we may write:

$$i\hbar\frac{\delta\psi(x,t)}{\delta t} = -\frac{\hbar^2}{2m}\frac{\delta^2\psi(x,t)}{\delta x^2} \quad (3)$$

Equation (3) is the Schrodinger equation for a free particle (where the potential energy V=0). We shall discuss the details in next sessions.

# References

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[1] Figures are collected from online resources.