Quantum Mechanics-Section16

Dr. Jayanta Das

Department of Physics, Maharaja College, Ara, Bihar-802301

1 Probabilistic interpretation of ψ

We should keep in mind that $\psi(x,t)$ is a complex function.

$$\psi(x,t) = \int dp\,\phi(p)\,e^{i(px-(p^2/2m)t)/\hbar}$$

and $\psi(x.t)$ is large where the particle is supposed to be and small everywhere else. Max Born studies the scattering of electrons from a target and suggested the proper interpretation of wave function. He proposed that

$$P(x,t)dx = |\psi(x,t)|^2 dx$$

defined that the probability that the particle described by the wave function $\psi(x,t)$ may be found between x and $x + dx$ at time t.

The probability density $P(x,t)$ is real, is large where the particle is supposed to be and its spreading does not imply that a particular particle is spreading. All it means is that as time goes by one is less likely to find the particle where one put it at $t = 0$.

For this interpretation to hold we must require that

$$\int_{-\infty}^{\infty} P(x,t)dx = 1$$

since the particle must be somewhere.

1.1 Probability current

We have already constructed the partial differential equation

$$i\hbar\frac{\delta\psi}{\delta t} = -\frac{\hbar^2}{2m}\frac{\delta^2\psi}{\delta x^2}$$

The complex conjugate of the above equation is

$$-i\hbar\frac{\delta\psi^*}{\delta t} = -\frac{\hbar^2}{2m}\frac{\delta^2\psi^*}{\delta x^2}$$

The probability of finding the particle at point x, and time t: $P(x,t) = \psi(x,t) * \psi(x,t)$ Differentiating with respect to t:

$$\frac{\delta}{\delta t}P(x,t) = \frac{\delta\psi^*}{\delta t}\psi + \frac{\delta\psi}{\delta t}\psi^*$$

$$\frac{\delta}{\delta t}P(x,t) = \frac{1}{i\hbar}\left(\frac{\hbar^2}{2m}\frac{\delta^2\psi^*}{\delta x^2}\psi - \frac{\hbar^2}{2m}\psi^*\frac{\delta^2\psi}{\delta x^2}\right)$$

$$\frac{\delta}{\delta t}P(x,t) = -\frac{\delta}{\delta x}\left(\frac{\hbar}{2im}(\psi^*\frac{\delta\psi}{\delta x} - \frac{\delta\psi^*}{\delta x}\psi)\right)$$

If we define the flux (or the probability current) by

$$j(x,t) = \frac{\hbar}{2im}\left(\psi^* \frac{\delta\psi}{\delta x} - \frac{\delta\psi^*}{\delta x}\psi\right)$$

We see that

$$\frac{\partial P(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0$$

The above equation is written in one dimensional form. The generalized form of the above equation may be expressed as:

$$\frac{\partial P(\mathbf{r},t)}{\partial t} + \nabla . j(\mathbf{r},t) = 0$$

where,

$$P(\mathbf{r},t) = |\psi(\mathbf{r},t)|^2$$

and

$$j(\mathbf{r},t) = \frac{\hbar}{2im}[\psi^*(\mathbf{r},t)\nabla\psi(\mathbf{r},t) - \psi(\mathbf{r},t)\nabla\psi^*(\mathbf{r},t)]$$

References

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[1] Figures are collected from online resources.