Quantum Mechanics-Section19

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1 Schrodinger Equation: Particle in a potential

The equation

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}$$

may be expressed with the identification $-i\hbar\frac{\partial}{\partial x} = \hat{p}$ in the following form:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \frac{\hat{p}^2}{2m}\psi(x,t)$$

The operator in the *rhs* is the of a free particle. If the particle is under a potential $V(x)$ then, we write

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left(\frac{\hat{p}^2}{2m} + V(x)\right)\psi(x,t)$$

therefore,

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

This is the basic equation of non-relativistic quantum mechanics and was proposed by Schrodinger. The Schrodinger equation can be written in other form:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \hat{H}\psi(x,t)$$

, where $\hat{H}$ is energy operator, called the Hamiltonian operator. An operator whose expectation values for all admissible wave functions are real is called Hermitian operator. We shall see that $\hat{p}$ and $\hat{x}$ are Hermitian operators and for real $\hat{V}(x)$ so is

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$$

As we know the *rhs* of the above equation is the total energy E of the particle. Therefore, the Schrodinger equation may written in a simplified form as

$$\hat{H}\psi(x,t) = E\psi(x,t)$$

References

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[1] Figures are collected from online resources.