B. Sc. Math. Hons. Part III 7th paper

Differential Equations

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Standard forms

Standard form I

It is of the form $\boxed{f(p,q)=0}$

i.e. only p and q are present.

Method of solving will be explained in the following sums.

Ex. 1 Solve $pq = k$ (constant).

Soln. Given, $pq = k$ ——— (1)

It is of the form $f(p,q)=0$.

Its soln is given by

$$z = ax + by + c \quad (2), \quad \text{where } f(a,b)=0$$

$f(a,b)=0 \Rightarrow ab = k \Rightarrow b = \frac{k}{a}$

$$(2) \Rightarrow z = ax + \frac{k}{a}y + c \quad \text{——— (3)}$$

This is the complete integral.

FOR SINGULAR SOLUTION

Differentiating (3) partially with respect to a,

$$0 = x - \frac{k}{a^2}y \quad \text{——— (4)}$$

Differentiating (3) partially with respect to c, we get

$0 = 0 + 0 + 1 \Rightarrow 0 = 1$ which is meaningless.

So, there is no singular solution.

FOR GENERAL SOLUTION:

From (3), $z = ax + \frac{k}{a}y + c$

Put $c = \phi(a)$

$\Rightarrow z = ax + \frac{k}{a}y + \phi(a)$ —— (4)

Differentiating it partially with respect to a, we get

$0 = x - \frac{k}{a^2}y + \phi'(a)$ —— (5)

Elimination of ~~a~~ a from (4) and (5) gives the general integral (or solution).

Q 2 Solve $p^2+q^2=m^2$; where m is constant.

Soln Given, $p^2+q^2=m^2$ —— (1)

It is of the form $f(p,q)=0$.

$\therefore$ Its soln is given by

$z = ax + by + c$ | where $f(a,b)=0$

i.e. $a^2+b^2=m^2$

$\Rightarrow b=\sqrt{m^2-a^2}$

$\Rightarrow z = ax+\sqrt{m^2-a^2}\,y+c$ —— (2)

This is the complete integral.

Singular soln Differentiating (2) partially with respect to a, we get

$$0 = x + \frac{1}{2\sqrt{m^2-a^2}}\times(-2a)y$$

$$\Rightarrow 0 = x - \frac{ay}{\sqrt{m^2-a^2}}$$

Differentiating (2) partially with respect to c, we get

$0 = 0+0+1 \Rightarrow 0=1$ which is absurd.

So, there is no singular soln.

General soln. from (2) $z=ax+\sqrt{m^2-a^2}\,y+c$

Put $c=\phi(a)$

$\Rightarrow z = ax+\sqrt{m^2-a^2}\,y+\phi(a)$ —— (3)

Differentiating (3) partially w.r.t a, we get.

$$0 = x + \frac{1}{2\sqrt{m^2-a^2}}\times(-2a)y+\phi'(a) \quad \text{—— (4)}$$

Elimination of a from (3) and (4) gives the general soln.

Q. 3 Solve $p+q=pq$.

Soln: Given, $p+q=pq$ —— (1)

Complete integral

It is of the form $f(p,q)=0$.

Its soln. is given by

$z = ax + by + c$ where $f(a,b)=0$

i.e. $a+b=ab$

$\Rightarrow a = ab-b = b(a-1)$

$\Rightarrow b = \frac{a}{a-1}$

$\Rightarrow z = ax + \frac{a}{a-1}y + c$ —— (2)

This is the C.I.

Singular soln.

Diff (2) partially with respect to a, we get

$0 = x + y\left[\frac{1}{a-1} - \frac{a}{(a-1)^2}\right]$.

Differentiating (2) partially w.r.to c, we have

$0 = 0+0+1 \Rightarrow 0=1$ which is absurd.

So, there is no singular soln.

General soln.

From (2), $z = ax + \frac{a}{a-1}y + c$

Put $c = \phi(a)$

$\Rightarrow z = ax + \frac{a}{a-1}y + \phi(a)$ —— (3)

Diff (3) partially w.r.to a, we get

$0 = x + y\left[\frac{1}{a-1} - \frac{a}{(a-1)^2}\right]y + \phi'(a)$ —— (4)

Elimination of a from (3) and (4) gives the required general soln.