B.Sc. Physics Part-II

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Paper - IV

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Classical Electrostatics: Static charges create static electric fields.

Classical Magnetostatics: steadily moving charges create static magnetic field

Classical Electrodynamics (Maxwell Equations): Accelerating charges create changing fields

Classical means here — Macroscopic theory

Coulomb's law:

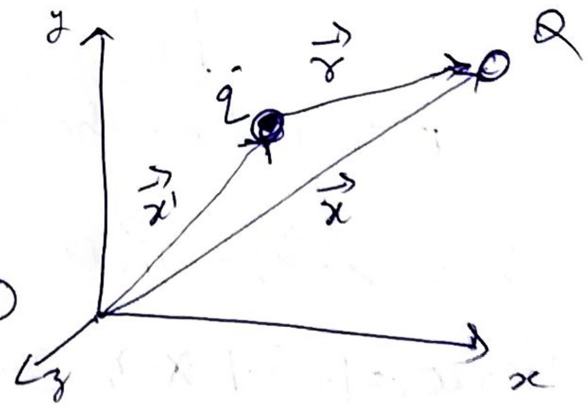
Force (F) $\propto qQ$

$F \propto \frac{1}{r^2}$

$F \propto \frac{qQ}{r^2}$

or $\vec{F} \propto \frac{qQ}{r^2}\hat{r}$

$$\boxed{\vec{r} = \vec{x} - \vec{x}'}$$



or $$\boxed{\vec{F} = k\frac{qQ}{r^2}\hat{r}} \quad \text{— (1)}$$

$k = \frac{1}{4\pi\epsilon_0}$ (In SI unit) (in free space)

Coulomb's law

$$\vec{F} = k \frac{qQ}{r^2} \frac{\vec{r}}{r}$$

or
$$\boxed{\vec{F} = k \frac{qQ \vec{r}}{r^3}} \quad (2)$$

$, \vec{r} = \vec{x} - \vec{x}',$

$r = |\vec{r}| = |\vec{x} - \vec{x}'|$

Eq. (1) & (2) both are same.

$$\vec{F} = Q \cdot \vec{E} = Q \cdot \frac{k q \vec{r}}{r^3}$$

$$\boxed{\vec{E} = k \frac{q \vec{r}}{r3}} \quad \text{or} \quad \boxed{\vec{E} = k \frac{q(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}}$$

For n point charges $\{q_i\} = q_1, q_2, \ldots q_n$

$$\boxed{\vec{E} = k \sum_{i=1}^{n} q_i \frac{(\vec{x} - \vec{x}_i)}{|\vec{x} - \vec{x}_i|^3}}$$

Note: ~~Before~~ (i) Since electric field is a vector field, it is defined at every point in space and therefore can be written in terms of spatial coordinates.

(ii) Force is not a field of vectors. This is the force felt at the point of particular charge q_i.

Before moving to notion of ~~char~~ continuous charge distribution we review here the Dirac Delta function.

One Dimensional Delta function

We denote it by $\delta(x)$ in 1D.

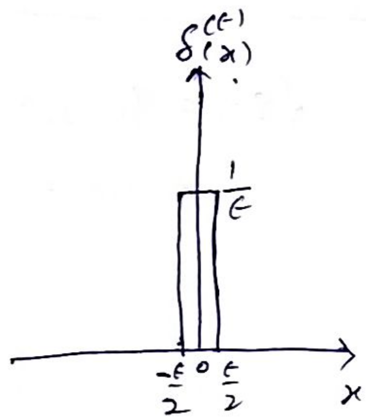
There are various definitions of the delta function.

ⓐ Delta function can be defined as the limit of $\delta^{(\epsilon)}(x)$ when $\epsilon \to 0$ as {see Fig.}

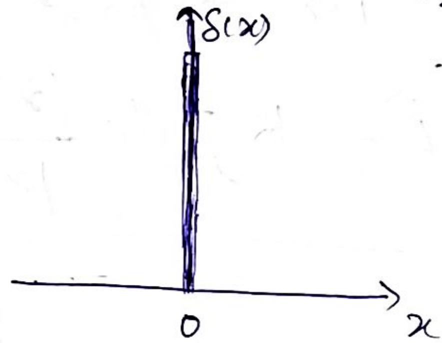
$$\delta(x) = \lim_{\epsilon \to 0} \delta^{(\epsilon)}(x)$$

where

$$\delta^{(\epsilon)}(x) = \begin{cases} \frac{1}{\epsilon}, & -\frac{\epsilon}{2} < x < \epsilon/2 \\ 0, & |x| > \epsilon/2 \end{cases}$$



$\delta^{(\epsilon)}(x)$



$\delta(x)$ {here $\epsilon \to 0$}

ⓑ $\delta(x)$ can also be defined using following integral equations.

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x)\,dx = f(0)$$

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x-a)\,dx = f(a)$$

→ δ-function is not a function in usual mathematical sense.

→ It can be expressed as the limit of analytical functions such as

$$\delta(x) = \lim_{\epsilon \to 0} \frac{\sin(x/\epsilon)}{\pi x},$$

$$\delta(x) = \lim_{a \to \infty} \frac{\sin^2(ax)}{\pi a x^2},$$

$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{x^2+\epsilon^2} \quad \text{etc}$$

~~Important properties~~:

Summary!

$$\delta(x) = \begin{cases} 0, & \text{if } x \neq 0 \\ \infty, & \text{if } x = 0 \end{cases}$$

and $\int_{-\infty}^{+\infty} \delta(x)\,dx = 1$.

Important properties:

(i) δ-function is an even function: $\delta(-x) = \delta(x)$

or $\delta(x-a) = \delta(a-x)$

(ii) $$\int_a^b f(x)\,\delta(x-x_0)\,dx = \begin{cases} f(x_0), & \text{if } a<x_0<b \\ 0, & \text{elsewhere.} \end{cases}$$

(iii) $\delta(x) = 0$, for $x \neq 0$.

(iv) $x\,\delta(x) = 0$.

(v) $\delta(ax) = \frac{1}{|a|}\delta(x)$, for $a \neq 0$

(vi) $f(x)\,\delta(x-a) = f(a)\,\delta(x-a)$

(vii) $\int_c^d \delta(a-x)\,\delta(x-b)\,dx = \delta(a-b)$ for $c \le a \le d,\ c \le b \le d$,

(viii) $\int_a^b \delta(x)\,dx = 1$ for $a \le 0 \le b$

(ix) $$\delta[g(x)] = \sum_i \frac{1}{|g'(x_{i})|}\,\delta(x-x_i)$$

$x_i \rightarrow$ zero of $g(x)$ and $g'(x_0) \neq 0$.

e.g, (a) $\delta(x^2 - a^2)$

here $g(x) = x^2 - a^2$

$$x^2 - a^2 = 0$$
$$\Rightarrow x = \pm a$$

$$g'(x) = 2x$$
$$g'(x)\big|_{x=+a} = 2a$$
$$g'(x)\big|_{x=-a} = -2a$$

$$\therefore\ \delta(x^2 - a^2) = \sum_0 \frac{1}{|g'(x_0)|}\,\delta(x - x_0)$$

$$= \sum_{x_0=+a} \frac{1}{|g'(x_0)|}\,\delta(x-x_0) + \sum_{x_0=-a} \frac{1}{|g'(x_0)|}\,\delta(x-x_0)$$

$$= \frac{1}{|2a|}\,\delta(x-a) + \frac{1}{|-2a|}\,\delta(x+a)$$

$$\boxed{\delta(x^2 - a^2) = \frac{1}{2|a|}\left[\delta(x-a) + \delta(x+a)\right]}$$

H.W. Obtain expression for $\delta[(x-a)(x-b)]$ following the above example.