TOPOLOGY FOR M.Sc. sem II (9th paper)

Unit 4

08/7

connectedness :

Let X be a topological space. Then X is said to be dis-connected if there exist two NON-EMPTY, DISJOINT OPENSETS G and H of X such that $X = G \cup H$.

Now, $X = G \cup H$, and

G and H are disjoint $\Rightarrow G \cap H = \phi$

So, $G = H^C$ and $H = G^C$

$\Rightarrow$ If G and H are open, they are closed also.

Thus, a topological space X is said to be connected if X is not dis connected.

Theorem A subset Y of a topo. space X is disconnected iff Y is contained in the union of two open subsets G, H of X whose intersections with Y are non-empty and disjoint.

Proof

$$Y \cap (G \cup H) = (Y \cap G) \cup (Y \cap H)$$

$$\therefore Y \subseteq G \cup H \iff Y = (Y \cap G) \cup (Y \cap H) \quad \text{---(1)}$$

Necessary part

Let Y be disconnected.

$\Rightarrow$ Y is union of two non-empty disjoint open sets, sets which are open for induced topology.

$\because$ An open set of the induced topology is of the form $Y \cap G$ where G is open in X.

$\Rightarrow$ $\exists$ two open subsets G, H of X such that

$$Y = (Y \cap G) \cup (Y \cap H) \quad \text{---(2)}$$

such that $Y \cap G$ and $Y \cap H$ are non-empty and disjoint

$(2) \Rightarrow Y \subseteq G \cup H$. Proved

Sufficient part To prove Y is ~~disconnected~~ disconnected.

from (1) $Y = (Y \cap G) \cup (Y \cap H)$ ---(3)

$\Rightarrow$ (3) is a disconnection of Y.

$\Rightarrow Y$ is disconnected.