M.Sc. Sem I. 1st Paper

Algebra (finite Field Extension)

Theorem: If L is a finite extension of F and K is a subfield of L which contains F, then $[K:F]$ is a divisor of $[L:F]$.

Soln: Let F, K, L be the given fields such that $F \subseteq K \subseteq L$.

$\because$ L is a finite extension of F

$\Rightarrow$ $L(F)$ is finite dimensional.

Let S be a finite basis of $L(F)$.

$\Rightarrow$ S generates $L(F)$.

Now, $F \subseteq K$. $\Rightarrow$ S generates $L(K)$.

$\Rightarrow$ $L(K)$ is finitely generated and therefore, a finite dimensional space.

$\because$ Every subspace of a finite dimensional vector space is finite dimensional

$\Rightarrow$ $K(F)$ is finite dimensional subspace of $L(F)$.

So, $[L:K]$, $[L:F]$ and $[K,F]$ all are finite.

By, transitivity of finite extensions,

$$[L:F] = [L:K][K:F]$$

$\Rightarrow$ $[K:F]$ is a divisor of $[L:F]$.

Proved

$\dot{\theta}^2$

SIMPLE FIELD EXTENSION

Let K be a field. Let $S \subseteq K$. Then the smallest subfield of K containing S is said to be generated by S.

Let K be an extension of a field F and xed $S \subseteq K$. Then, the subfield of K generated by $F \cup S$ is called the subfield of K generated by S over F, denoted as $F(S)$. shows

If $\alpha \in K$, the subfield of K generated by $F \cup \{\alpha\}$ is called a subfield of K generated by α over F. It is denoted as $F(\alpha)$.

Thus, $F(\alpha)$ is the field obtained by adjoining α to F and this process of obtaining $F(\alpha)$ is called an ADJUNCTION of α to F.

* Now, we define <u>simple extension of a field</u>:

The extension K of a field F is called a simple extension of F if

$$K = F(\alpha) \text{ for some } \alpha \in K.$$

α is called a <u>PRIMITIVE ELEMENT</u> of K over F.

* <u>ROOT OF A POLYNOMIAL</u>

Let K be an extension of a field F. Then, an element $\alpha \in K$ is called a root of the polynomial $f(x)$ in $F[x]$ if $f(\alpha) = 0$.