M.Sc. (Maths). Sem. II | 9th Paper CC-09

TOPOLOGY (CONTINUED)

UNIT 5

10/7/2020

Regular spaces

Let X be a topological space. Let $x \in X$. Let F be a closed subset of X.

Then X is called a regular space if for every closed subset F of X and $\forall\, x \in X$ such that $x \notin F$, there exist two disjoint open sets G and H such that

$$F \subseteq G \text{ and } x \in H.$$

Example Let $X = \{x, y, z\}$,

and $T = \{\phi, X, \{x\}, \{y, z\}\}$.

obviously, T is a topology on X because

(T_1) $X, \phi, \in T$

(T_2) $\phi \cup X = X \in T$, $X \cup \{x\} = X \in T$

$\{x\} \cup \{y,z\} = \{x,y,z\} = X \in T$

Similarly others

(T_3) $\phi \cap X = \phi \in T$

$X \cap \{x\} = \{x\} \in T$

$X \cap \{y,z\} = \{y,z\} \in T$

$\{x\} \cap \{y,z\} = \phi \in T$

Similarly others.

closed subsets of topo. space (X,T)

$= X, \phi, \{y,z\}, \{x\}.$

Now for closed subset $F = \{y,z\}$ of X and point $x \in X$ $(x \notin \{y,z\})$, there exist disjoint open sets :

$G = \{y,z\}$ and $H = \{x\}$ such that

$F \subseteq G$ and $x \in H$.

So, (X,T) is a regular space.

T_3-SPACE

Every regular T_1-space is called T_3-space.

$\boxed{Q:}$ Prove that every regular space is not a T_1-space.

<u>Proof</u> Let $X = \{a, b, c\}$,

and $T = \{\phi, X, \{a\}, \{b, c\}\}$

As earlier, we can show that T is a topology on X. Also, it is easy to verify that (X, T) is a regular space.

Let us recall the definition of T_1-space:

A topo. space (X, T) is said to be <u>T_1-space</u> iff for any given pair of distinct points x and y of X

there exist two open sets, one containing x but not y and another containing y but not x.

Also, another definition of T_1-space :→

X is a T_1-space iff every singleton subset of X is a closed set.

Here $X = \{a, b, c\}$ and

$T = \{\phi, X, \{a\}, \{b, c\}\}$

Here, the singleton set $\{c\}$ is not closed.

Hence (X, T) is not a T_1-space.