B.Sc. Part III (Hons.) 5th Paper

Infinite series

Infinite Products (contd.)

Q. Test the convergence of $\prod_{1}^{\infty}\left(1+\frac{1}{n^{3/2}}\right)$.

Soln Let the given infinite product be denoted by $\prod_{1}^{\infty}(1+U_n)$.

$$\Rightarrow \prod_{1}^{\infty}(1+U_n) = \prod_{1}^{\infty}\left(1+\frac{1}{n^{3/2}}\right)$$

$$\therefore U_n = \frac{1}{n^{3/2}},$$

$$\Rightarrow \sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$$ which is convergent

as $p = 3/2 > 1$.

$$\Rightarrow \prod_{1}^{\infty}(1+U_n)$$ is convergent.

$\prod_{1}^{\infty}(1 \pm U_n)$ converges or diverges according as the series $\sum U_n$ converges or diverges.

Q. Test the convergence of the product $\prod_{1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)$.

Solution

Here $\prod_{1}(1+U_n) = \prod_{1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)$

$\Rightarrow \quad U_n = \frac{1}{\sqrt{n}}$

$\Rightarrow \quad \sum U_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$

But $\sum \frac{1}{\sqrt{n}}$ (or $\sum \frac{1}{n^{1/2}}$) is divergent

as $p = \frac{1}{2} < 1$.

$\Rightarrow \quad \prod_{1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)$ is divergent.

Q. Test the convergence of the product $\prod_{1}^{\infty}\left(1-\frac{1}{n+1}\right)$

Soln. Here the infinite product

$= \prod_{1}^{\infty}(1-U_n) = \prod_{1}^{\infty}\left(1-\frac{1}{n+1}\right)$

$\Rightarrow \quad U_n = \frac{1}{n+1} \Rightarrow \sum U_n = \frac{1}{2}+\frac{1}{3}+\cdots$

$= \sum \frac{1}{n+1}$ which

is divergent. $\Rightarrow$ the given infinite product is also divergent.