B.Sc. Part-III, Paper-V, Group-A

email: sunil.phy30@gmail.com

Partial differential equations:

Differential equations in one variable → Ordinary differential equations (ODEs)

Two or more variables → partial differential equations (PDEs)

Examples of partial differential equations:–

Most encountered PDEs in physics

(i) Laplace's equation: $\nabla^2 u = 0$

Function u may be – gravitational potential in a region containing no matter

– Electrostatic potential in a charge-free region

– Steady-State temperature

– Velocity field for an incompressible fluid

(ii) Poisson's equation: $\nabla^2 u = f(x, y, z)$

In contrast to the homogeneous Laplace's equation the function u may represent the same physical quantities as discussed above but in a region which contains matter, electric charge, heat sources, fluid respectively $f(x, y, z)$ is source term.

(iii) Helmholtz equations: $\nabla^2 u \pm k^2 u = 0$

~~These equations~~

$u \to$ time independent

These equation appear in describing the physics of

- elastic waves in solids
- electromagnetic waves
- sound or acoustics
- nuclear reactors

(iv) Time-dependent diffusion equation: $\nabla^2 u = \frac{1}{a^2}\frac{\partial u}{\partial t}$

Here u may be ~~more~~ non-steady state temperature in a region with no heat sources or concentration of a diffusing substance.

$\alpha^2 \to$ constant called diffusivity

(v) wave equation: $\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$

$v \to$ speed

(vi) Schrödinger wave equation:

Time-dependent $\to$ $i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi$

$\to$ time independent $\to$ $\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$