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Differential Equations (Formation & Solution)

Def. (Diff. Eqn) :- A equation involving the dependent variable and independent variable and also the derivatives of the dependable variable is known as a differential equation.

Differential equation is formed from a given equation say in x and y.

Illustrations of the formation of diff. eqn by elimination.

(i) Let $y = mx + m^2$ ——— ①

be an equation containing x and y and one arbitrary constant m.

Differentiating with respect to x, we get

$$\frac{dy}{dx} = m(1) + 0$$

$$\Rightarrow \frac{dy}{dx} = m \quad \text{———} \quad ②$$

By Eliminating m betn. ① and ②, we have

$$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

which gives us a diff. eqn involving x, y and $\frac{dy}{dx}$

(ii) Let $y = A\cos x + B\sin x$ ——— ①

be an eqn. containing x and y and two arbitrary constants A and B.

Diff. w.r.t. x, we get

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

Again, Diff. w.r.t. x, we have

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x = -(A\cos x + B\sin x) = -y \quad [\text{from}(1)]$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = \cancel{-A\cos x} \cancel{- B\sin x} + \cancel{A\cos x} + \cancel{B\sin x} = 0$$

or, $$\frac{d^2y}{dx^2} + y = 0.$$

This gives us a diff. eqn connecting y and the second order differential coefficient $\frac{d^2y}{dx^2}$.

Some of the other examples of differential eqns. are the following:-

(1) $\frac{dy}{dx} = \sin x$ (2) $\frac{dy}{dx} + 3y = e^{2x}$ (3) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2$ etc.

A Solution of a diff. eqn. is a relation between the variables (not involving the differential coefficients) from which the given diff. eqn. can be derived by the process of differentiation and other algebraic processes of elimination etc.

Ex. Form the diff. eqn. for the curve $y^2 = 4a(x+a)$.

Soln. Given $y^2 = 4a(x+a) = 4ax + 4a^2$, ——— (1)

By differentiating, $2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$ ——— (2)

By eliminating a from (1) and (2), we get

$$y^2 = 4x\frac{1}{2}y\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$