

$$x \frac{dy}{dx} - y = x^4 - 3x$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x^3 - 3 \quad \text{--- (1)}$$

This is a linear diff. eqn of the form  $\frac{dy}{dx} + Py = Q$ .

$$\text{Here, } P = -\frac{1}{x}, Q = x^3 - 3$$

$$\therefore \text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

Multiplying the equation by IF and integrating we get

$$\frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = \frac{1}{x} (x^3 - 3)$$

$$\Rightarrow \frac{1}{x} \cdot \frac{d}{dx}(y) + y \cdot \frac{d}{dx}\left(\frac{1}{x}\right) = x^2 - \frac{3}{x}$$

$$\Rightarrow d\left(\frac{y}{x}\right) = x^2 - \frac{3}{x}$$

Integrating, we have

$$\frac{y}{x} = \int \left(x^2 - \frac{3}{x}\right) dx \Rightarrow \boxed{\frac{y}{x} = \frac{x^3}{3} - 3 \log x + C}$$