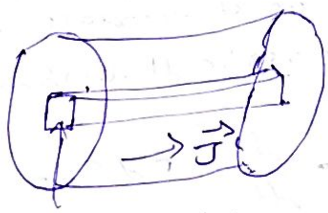
Volume current density ($\vec{J}$).

When we consider the distribution of charges over three-dimensional region, we need to introduce the concept of Volume current density ($\vec{J}$).

In fig. we have ~~con~~ taken a tube of infinitesimal cross section $da_\perp$. which runs parallel to the flow of the current. Let $d\vec{I}$ denote the current in the tube.



$da_\perp$

Now the volume current density $\vec{J}$ is given by

$$\boxed{\vec{J} \equiv \frac{d\vec{I}}{da_\perp}} \equiv \text{current per unit area perpendicular to flow} \quad \text{—(i)}$$

If we denote volume charge density by ρ and its velocity by $\vec{v}$, then $\vec{J}$ is given by

$$\boxed{\vec{J} = \rho\vec{v}} \quad \text{—(ii)}$$

Next, the magnetic force on a volume current is given as

$$\vec{F}_m = \int(\vec{v}\times\vec{B})\rho\, d\tau = \int(\vec{J}\times\vec{B})\, d\tau \quad \text{—(iii)}$$

Continuity equation:

From eq. (i) the current crossing the surface S can be written as

$$I = \int_S J\, da_\perp = \int \vec{J}\cdot d\vec{a}$$

Next, the total charge per unit time leaving a volume V is given as

$$\oint_S \vec{J} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{J})\, d\tau$$

Since we know that charge is conserved. Charge in one region must decrease as a result of the flow of current out of the region through the cross sectional area. We, therefore, write

$$-\frac{d}{dt}\int_V \rho\, d\tau = \int_V (\nabla \cdot \vec{J})\, d\tau$$

or

$$-\int_V \left(\frac{\partial \rho}{\partial t}\right) d\tau = \int_V (\nabla \cdot \vec{J})\, d\tau$$

Since above expression is true for any arbitrary volume, thus we obtain

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0}$$

The above equation is called the continuity equation.

~~This is the~~

This equation signifies the conservation of charge.

Q. Suppose that the magnetic field in some region has the form $\vec{B} = kz\,\hat{x}$ (k = constant)

Find the force on a square loop (side a), lying in the yz plane and centered at the origin, if it carries a current I, flowing counterclockwise, when you look down the x axis