Galilean Transformation-II

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"You cannot teach a man anything, you can only help him discover it in himself."
— Galileo Galilei (1564-1642)

Frame of reference

Here we discuss basic definition of reference frame with examples. A frame of reference is a set of coordinates which are used to specify an *event* [∗] such as turning on a flashlight, collision of two objects, etc. There are two categories of reference frames: Inertial and Non-inertial.

A. Inertial reference frame

- An inertial system can be treated as a frame of in which *Newton's first law which is also known as the law of inertial*, holds. These frames are called as *inertial reference reference.*
- An unaccelerated system, i.e., a system *moving with constant velocity* with respect to an inertial frame is also called as inertial reference frame.
- Some more examples of inertial reference framea are as follow:
  - A set of axes fixed on earth[†] is treated as the inertial coordinate system.

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[∗] An event is described by a point in space-time, i.e., (x,y,z,t).

[†] There is small acceleration effect due to rotation (around its axis) and orbital motion of the earth around the sun. Generally, this small contribution to acceleration is neglected and earth is treated as the inertial system.

– Any set of axes moving with constant velocity with respect to earth is treated as inertial frame. For example, a train/car/motorcycle/boat/ship/helicopter/airplane moving with constant velocity with respect to ground is inertial reference frame.

I. GALILEAN TRANSFORMATION

In Fig. (1) we have denoted an event with a point P. Space and time coordinates in the two frames (S and S') are related by the following transformation rule (see Fig (1).)

$$\begin{aligned} x' &= x - ut, \\ y' &= y, \\ z' &= z, \\ t' &= t\ . \end{aligned} \tag{1}$$

Note that coordinates of the S'-frame are denoted with a prime on them to distinguish with the corresponding coordinates of the S-frame. Now using vectorial notation, we can write the above equations in compact form as

$$\mathbf{x}' = \mathbf{x} - \mathbf{u}t; \quad t' = t\ , \tag{2}$$

where $\mathbf{u} = (u_x = u, u_y = 0, u_z = 0)$ as defined in Fig. (1). We can write Eq. (2) in matrix form as

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - t \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}; \quad t' = t\ . \tag{3}$$

A. Time and space interval under Galilean transformation

1. *Time interval between two events*

Time interval between two events A and B are invariant under Galilean transformation, i.e., interval is same for both the observers.

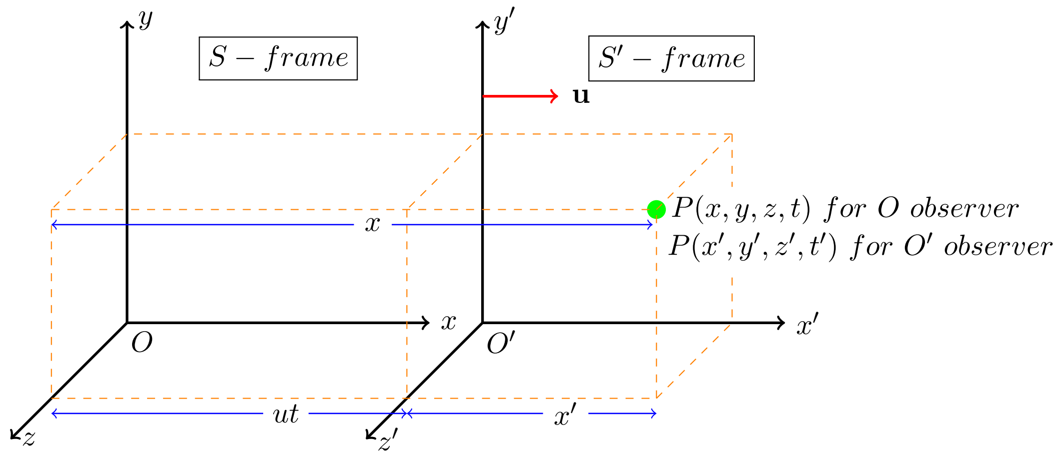

FIG. 1: S' frame moving with constant velocity $\mathbf{u}$ relative to S frame in the +ive $x-$direction, where $\mathbf{u} = (u, 0, 0)$. Both the frames have common x-x'-axis and other axes y-y' and z-z' of the two frames are parallel. Initially at $t = t' = 0$, both the frames were coinciding at the origin O. After some time t, the $S' - frame$ moved to a distance ut in the +ive $x-$direction.

From Eq. (1), we can write

$$t'_A - t'_B = t_A - t_B. \tag{4}$$

Above expression signifies that time interval is invariant.

2. Length

Length, (distance or space interval) between two points P and Q is invariant under Galilean transformation, i.e., the distance between the two points measured at a given instant is same for both the observers.

To prove this using Galilean transformation rule given by Eq. (1), we consider a rod which is at rest in S frame and whose ends points are P and Q. Using Eq. (1) we can write

$$x'_Q - x'_P = x_Q - x_P - u(t_Q - t_P), \tag{5}$$

where $x'_Q - x'_P$ denotes the length of rod for the S' observer who observes that rod is moving with velocity $-u$. Since the end points of the rod P and Q are measured at the same time so we can write $t_Q = t_P$. Now we obtain Eq. (5)

$$x'_Q - x'_P = x_Q - x_P, \tag{6}$$

which shows that length is invariant.

B. Velocity addition theorem

Differentiating the space transformation rules of Eqs. (1) with respect to (wrt) t we obtain,

$$\begin{aligned} \frac{dx'}{dt'} &= \frac{dx}{dt} - u, \\ \frac{dy'}{dt'} &= \frac{dy}{dt}, \\ \frac{dz'}{dt'} &= \frac{dz}{dt}. \end{aligned} \tag{7}$$

In above Eqs. we have taken $d/dt \equiv d/dt'$ since $t' = t$. Next, we define $dx/dt = v_x$, $dy/dt = v_y$, and $dz/dt = v_z$ and similarly we take corresponding velocity components in the primed frame. Now we obtain the velocity addition theorem (classical)as

$$\begin{aligned} v'_x &= v_x - u, \\ v'_y &= v_y, \\ v'_z &= v_z. \end{aligned} \tag{8}$$

Vectorial form of above equations can be written as

$$\mathbf{v}' = \mathbf{v} - \mathbf{u}. \tag{9}$$

The above equation relates the object velocity in the two frames.

C. Acceleration transformation rule

Here we obtain the acceleration transformation rule in the moving frame and the rest frame. We differentiate Eq. (7) wrt t Differentiating the space transformation rules of Eqs. (1) with respect to (wrt) t we obtain,

$$\begin{aligned} \frac{dv'_x}{dt'} &= \frac{dv_x}{dt} \Rightarrow a'_x = a_x, \\ \frac{dv'_y}{dt'} &= \frac{dv_y}{dt} \Rightarrow a'_y = a_y, \\ \frac{dv'_z}{dt'} &= \frac{dv_z}{dt} \Rightarrow a'_z = a_z. \end{aligned} \tag{10}$$

Time derivative u is zero since it is constant. From the above equation we see that acceleration components remain unchanged hence acceleration, i.e., $\mathbf{a}' = \mathbf{a}$, under Galilean transformation rule. This implies that acceleration of a particle remains same in all reference frames moving with constant speed relative to one another.

Note:

- The above result shows that Newton's laws of motion, equations of motion of a particle, conservation principles of mechanics such as linear momentum, angular momentum, energy are same in all inertial frames.
- In conclusion, we can say that **laws of mechanics are invariant under Galilean transformation**.

[1] R. Resnick, *Introduction to Special Relativity*, Wiley-VCH, (1968) .

[2] A. Beiser, *Concepts of modern physics*, Tata McGraw-Hill Education, (2003).