Work done in Reversible Isothermal Expansion:—

During Isothermal expansion, temperature of the gas remains Constant, Pressure decreases and Volume increases, these two Parameters Pressure & Volume are assigned Opposite Signs.

So the work done by the gas in an infinitesimal expansion is given by

$$dw = -(P - dP)\,dv$$
$$= -P\,dv + dP.dv \quad [\because dP \& dv \text{ are infinitesimal so its product may be neglected}]$$
$$= -P\,dv \quad \text{——— (1)}$$

The total work done by the gas,

$$W = -\int_{V_1}^{V_2} P\,dv$$

[where V_1 = initial volume, V_2 = final volume]

For One mole of gas, the gas eqn $PV = RT$

$$\text{or } P = \frac{RT}{V}$$

On Substituting the value of P in the above equation, we have-

$$W = -\int_{V_1}^{V_2} \frac{RT}{V}\,dv = -\int_{V_1}^{V_2} RT \cdot \frac{dv}{V}$$

$$W = -RT \int_{V_1}^{V_2} \frac{dv}{V} = -RT \ln \frac{V_2}{V_1} \quad \text{——— (2)}$$

Since In case of ideal gas, at Constant temperature $P_1V_1 = P_2V_2$

$$\text{or } \frac{V_2}{V} = \frac{P_1}{P_2}$$

$$\therefore W = -RT \ln \frac{P_1}{P_2} \quad \text{——— (3)}$$

For n moles the above eqn Can be written as

$$W = -nRT \ln \frac{V_2}{V_1}$$

$$\text{or } W = -nRT \ln \frac{P_1}{P_2}$$

$$\text{i.e, } W = -nRT \ln \frac{V_2}{V_1} = -nRT \ln \frac{P_1}{P_2} \quad \text{——— (4)}$$

Since during expansion, V_2 is more than V_1 and P_2 is less than P_1 hence from equation (4) the work comes out to be negative in Conformity with the Convension used in this regard.