Postulates of Quantum-Mechanics

The classical mechanics holds good to explain the all Phenomenon and Properties of Macroscopic particles but it fails to explain microscopic world like electron, atom - - - etc.
Hence a new concept like wave mechanical treatment is required to explain all phenomenon and properties of microscopic world.
Now a new set of rules come in to existance to explain it; which is known as Postulates of quantum-mechanics.
Postulates for a system moving in one dimension i.e along x-co-ordinate are given below.

Postulate - 1. The Physical state of a system at time t is described by the wave function $\psi(x,t)$.

2. The wavefunction $\psi(x,t)$ and its first and second derivatives $\frac{\partial \psi(x,t)}{\partial x}$ and $\frac{\partial^2 \psi(x,t)}{\partial x^2}$ are continuous, finite and single valued for all the value of x.

And also the wave function $\psi(x,t)$ is normalised i.e $\int_{-\infty}^{+\infty} \psi^*(x,t)\, \psi(x,t)\, dx = 1$

3. A Physically observable quantity can be represented by a Hermitian Operator.
Operator $\hat{A}$ is said to be Hermitian if it satisfies the condition given below.

$$\int \psi_i^* \hat{A} \psi_j \, dx = \int \psi_j \, (\hat{A}\psi_i)^* \, dx$$

where ψ_i and ψ_j are the wavefunctions representing the physical states of the Quantum System like atom or molecule.

4. The allowed values of an observable A are the eigen values.

$$\hat{A}\psi_i = a_i \psi_i$$

Here $\hat{A}$ is the operator for the observable and ψ_i is an eigenfunction of $\hat{A}$ with eigen value a_i. i.e measurement of the observable A yields the eigen value a_i.

5. The average value $\langle A \rangle$ of an observable A corresponding to the operator $\hat{A}$ is obtained from the relation, $\langle A \rangle = \int_{-\infty}^{+\infty} \psi^* \hat{A} \psi \, dx$

where the function ψ is assumed to be normalized, when it fulfill the condition of normalisation.

The average value along x-coordinate is given as

$$\langle x \rangle = \int_{-\infty}^{+\infty} \psi^* \hat{x} \psi \, dx$$

6. The wave function $\psi(x,t)$ is a solution of the time-dependent Schrodinger eqn

$$\hat{H}\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t},$$

Postulate-7 → To every observable such as Position, momentum, energy --- etc in classical mechanics, there corresponds an Operator in Quantum mechanics.

Classical-Mechanical observables and their Corresponding Quantum Mechanical Operators

Observable		Operator	
Name	Symbol	Symbol	Operation
Position	x	$\hat{x}$	Multiplication by x
	y	$\hat{y}$	Multiplication by y
Momentum	P_x	$\hat{P}_x$	$\frac{h}{2\pi i}\frac{\partial}{\partial x} = -i\hbar\frac{\partial}{\partial x}$
	P	$\hat{P}$	$\frac{h}{2\pi i}\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)$ $= -i\hbar\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)$
Kinetic Energy	K	$\hat{K}$	$-\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$ $= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$
Potential Energy	$V(x)$	$\hat{V}(x)$	Multiplication by $V(x)$
Total Energy	E	$\hat{H}$	$-\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) + V(x,y,z)$ $= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) + V(x,y,z)$
Angular Momentum	$L_x = yP_z - zP_y$	$\hat{L}_x$	$-i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$
	$L_y = zP_x - xP_z$	$\hat{L}_y$	$-i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$
	$L_z = xP_y - yP_y$	$\hat{L}_z$	$-i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$

Operator: — In Quantum Mechanics the role of Operator is very important. An Operator, Operate a function and changes the nature of function.

It means a function goes under operation by an Operator and provides another function which is different from the Original function.

In Other words we can say an Operator is a symbol, giving instructions for transforming a given mathematical function into another function according to a defined rule.

Operators has no Physical meaning if it stands alone. It is more clear by the following example.

" $\frac{\partial}{\partial x}$ " is an operator which in itself has no Physical meaning, but if put before a function, it transforms the function into its first derivative with respect to x,

$$\text{i.e } \frac{\partial}{\partial x} x^2 = 2x$$

Here Operator '$\frac{\partial}{\partial x}$' transforms the function x^2 into another function $2x$.

If $\hat{A}$ denotes an Operator which transforms the function $f(x)$ into another function $g(x)$,

Then we can write it as,

$$\hat{A} f(x) = g(x).$$

For every Physically measurable property of a microscopic system such as Position, momentum, Energy... etc. in Classical mechanics, there corresponds an Operator in Quantum mechanics.

For example, The operator for Linear momentum in one dimension (along x-axis i.e P_x) is as follow

$$\hat{P}_x = \frac{h}{2\pi i} \frac{\partial}{\partial x}$$

$$= \frac{h \times i}{2\pi i \times i} \frac{\partial}{\partial x}$$

$$= \frac{ih}{2\pi i^2} \frac{\partial}{\partial x} \quad \left[i^2 = -1 \ \& \ \frac{h}{2\pi} = \hbar \right]$$

$$\text{So, } \hat{P}_x = -i\hbar \frac{\partial}{\partial x}$$

And the Operator for angular momentum along x-axis,

$$\hat{L}_x = \frac{h}{2\pi i} \left(y\frac{\partial}{\partial z} - \cancel{z} \frac{\partial}{\partial y} \right)$$

$$\hat{L}_x = -i\hbar \left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \right)$$

—x—

Addition of Operators: –

Addition of two operators gives a new operator defined as $(\hat{A}+\hat{B})f(x) = \hat{A}f(x) + \hat{B}f(x)$

Let operator $\hat{A}$ is $\frac{d}{dx}$ and $\hat{B}$ is 'x' respectively and function $f(x)$ is x^2.

then $\left(\frac{\partial}{\partial x}+x\right)(x)^2 = \frac{\partial x^2}{\partial x} + x^3$

$= 2x + x^3$

Substraction of Operators: –

The substraction of two operators gives a new operator defined as

$$(\hat{A}-\hat{B})f(x) = \hat{A}f(x) - \hat{B}f(x)$$

Let operator $\hat{A}$ is $\frac{d}{dx}$

and $\hat{B}$ is $\frac{d^2}{dx^2}$

& $f(x)$ is $\sin x$

then,

$$\left(\frac{d}{dx} - \frac{d^2}{dx^2}\right)\sin x = \frac{d\sin x}{dx} - \frac{d^2(\sin x)}{dx^2}$$

$$= \cos x - (-\sin x)$$

$$= \cos x + \sin x$$

— x —

Multiplication of Operators:-

Multiplication of two operators means operations by two operators, one after the another, the order of operation being from right to left.

For example,

$$\hat{A}\,\hat{B}\,f(x)$$

In the above case the function $f(x)$ is first operated by $\hat{B}$ to give a new function $g(x)$ then function $g(x)$ is operated by $\hat{A}$ to give $h(x)$ as the final function.

$$\hat{A}\,\hat{B}\,f(x) = \hat{A}\,[\hat{B}\,f(x)]$$
$$= \hat{A}\,[g(x)]$$
$$= h(x)$$

let $\hat{A}$ is $\frac{d}{dx}$

$\hat{B}$ is $\ln$

and $f(x)$ is x^2

then, $\hat{A}\,\hat{B}\,f(x) = \frac{d}{dx}\log(x^2)$

$$= \frac{d}{dx}[\log x^2]$$
$$= \frac{d}{dx}(2\log x)$$
$$= 2\frac{d}{dx}\log x$$
$$= 2 \times \frac{1}{x}$$
$$= \frac{2}{x}$$

—x—