Adiabatic equation of State:-

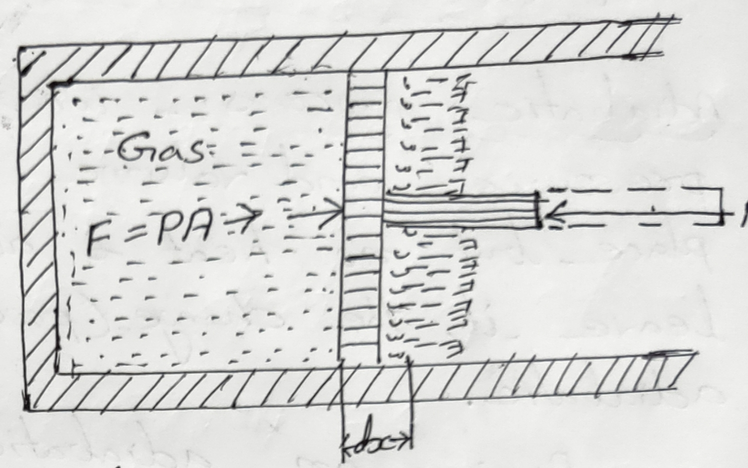

fig:-1

If we take one gram molecule of a perfe[ct] gas contained in a perfectly non-conducting cylinder fitted with a non-conducting piston as shown in fig1. where P, V & T are pressure, volume & temperature respectively. Suppose the gas is compressed adiabatically, so that the piston moves inwards through a distance dx. If A is the area of cross-section of the piston, then total force applied is $P \times A$ and

Work done by piston = Force × Distance

$= P \times A \times dx$

or $dW = PdV$ | As $Adx = dV$ = change in Volume.

The heat generated due to compression causes a rise of temperature dT and is given by $C_v dT$.

C_v = gram molecular specific heat at constant volume.

According to the first law of thermodynamics

$$dQ = dU + dW$$
$$= dU + PdV$$

Since for adiabatic change $dQ = 0$

$\therefore$ $dU + PdV = 0$

Now, $C_v = \frac{dU}{dT}$

$\therefore$ $dU = C_v dT$

Hence $C_v dT + PdV = 0$ ——— (i)

The equation of state for one mole of a perfect gas is $PV = RT$ ——— (ii)

After differentiating eqn (ii) we have

$$PdV = RdT - VdP$$

or $PdV + VdP = RdT$

$$\therefore \quad dT = \frac{PdV + VdP}{R}$$

putting value of dT in eqn (i) we have

$$C_V \frac{Pdv + Vdp}{R} + Pdv = 0$$

or, $C_V P dv + C_V V dp + RPdv = 0$

or, $(C_V + R) Pdv + C_V V dp = 0$

But $C_p - C_V = R$ or $C_V + R = C_p$

Hence $C_p P dv + C_V V dp = 0$

After Dividing by $C_V PV$ & putting $\frac{C_p}{C_V} = \gamma$ we have

where C_p = gram molecular Specific heat at constant pressure. & γ = is the ratio of sp. heat at constant pressure to sp. heat at const. volume.

$$\gamma \frac{dv}{V} + \frac{dP}{P} = 0$$

After integrating it

$$\gamma \log_e V + \log_e P = \text{constant}.$$

or $\log_e PV^\gamma = \text{constant}$

$$PV^\gamma = e^{\text{constant}} = \text{Constant} = K$$

Thus $PV^\gamma = \text{constant}$

This is the eqn of state for an adiabatic change (process) in terms of thermodynamic variables p & V.

If $P_1 V_1$ are the initial and $P_2 V_2$ the final pressure and volumes of the gas in an adiabatic change, then $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

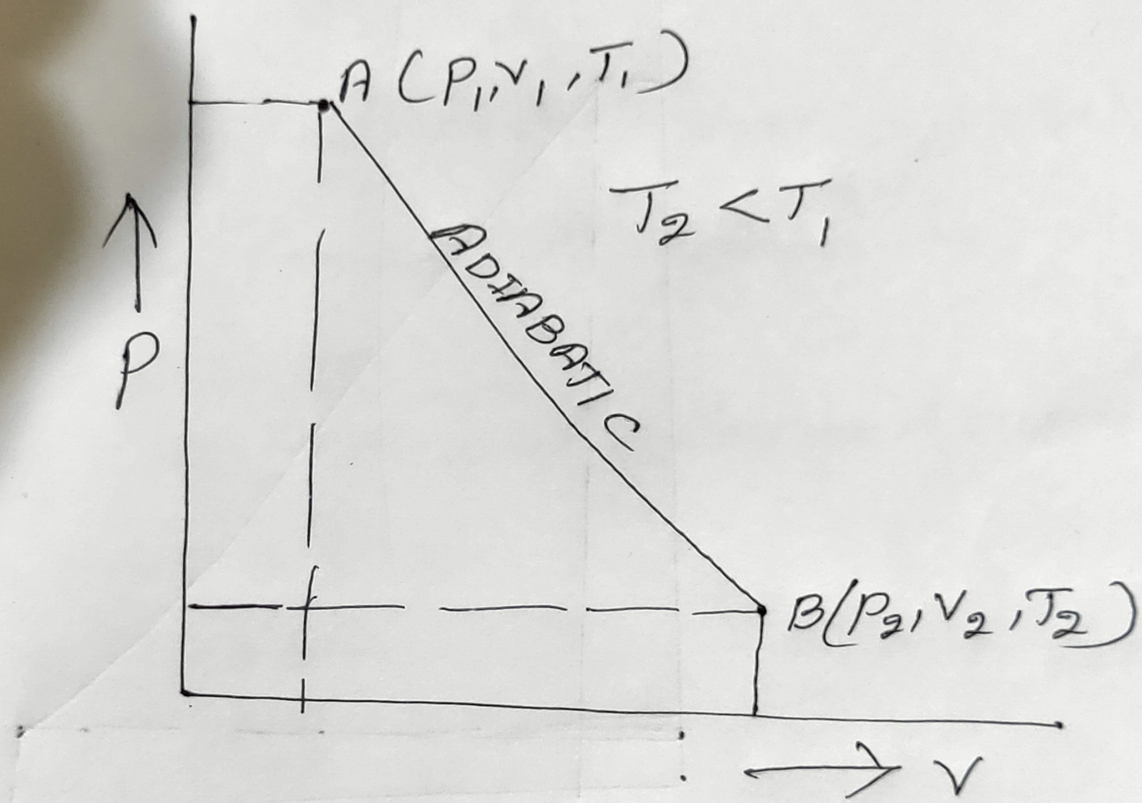

fig 2.

— x —