Electrical oscillations in an L-C Circuit

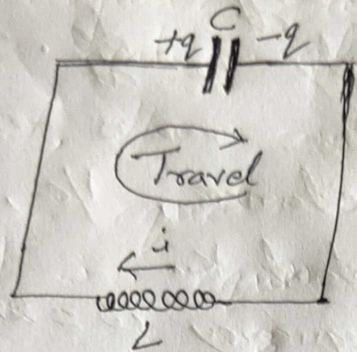

fig ①

fig① → Applying Kirchoff's loop rule to the L-C circuit. The direction of travel around the loop in the loop equation is shown. Starting at the lower-right corner of the circuit and adding voltages as we go clockwise arround the loop then we get

$$-L\frac{di}{dt} - \frac{q}{C} = 0 \quad \text{——— ①}$$

$$\because i = \frac{dq}{dt}, \therefore \frac{di}{dt} = \frac{d^2q}{dt^2}$$

putting this in equⁿ ① and divide by $-L$ to obtain

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \quad \text{——— ②}$$

But in simple harmonic motion

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x \quad \text{or}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

In an L-C circuit the capacitor charge q plays the role of the displacement x, and the current $i = \frac{dq}{dt}$ is analogous to the particle's velocity $v_x = \frac{dx}{dt}$. The inductance L is analogous to the mass m. and the reciprocal of capacitance, $1/c$, is analogous to the force constant K.

The angular frequency $\omega = 2\pi f$ of the harmonic oscillator is equal to $(K/m)^{\frac{1}{2}}$ and the position is given a function of time like

$$x = A \cos(\omega t + \phi)$$

where the amplitude A and the phase angle ϕ depend on the initial conditions. In the analogous electrical situation the capacitor charge q is given by

$$q = Q \cos(\omega t + \phi) \qquad (3)$$

and the angular frequency ω of oscillation is given by

$$\omega = \sqrt{\frac{1}{LC}} \qquad (4)$$

ω = Angular frequency of oscillation in an L-C circuit.

L = Inductance

C = Capacitance

then $$i = -\omega Q(\sin \omega t + Q) \qquad (5)$$ [from equn (3) & (4)]

where $i = \frac{dq}{dt}$.

Thus the charg & current in an L-C circuit oscillate sinusoidally with time, with an angular frequency determined by the values of L and C. The ordinary frequency f, the number of cycles per second, is equal to $\omega/2\pi$. The constants Q and ϕ in equn 3 and equn 5 are determined by the initial conditions. If at time $t=0$ the left-hand capacitor plate in fig 1 has its maximum charge Q and the current i is zero, then $\phi=0$. If $q=0$ at $t=0$, then $\phi=\pm\pi/2$ rad.

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