Relation betn Adiabatic & Isothermal Elasticity

Volume elasticity, E is defined as

$$E = -\frac{dp}{dV/V}$$

where dp = applied pressure (stress)

& $-\frac{dV}{V} = \frac{\text{Change in volume}}{\text{original volume}}$ = Volumetric strain.

∵ For an isothermal change,

pV = a constant.

After differentiating, we get $pdV + Vdp = 0$

or, $p = -V\frac{dp}{dV} = -\frac{dp}{dV/V} = E_{iso}$

For an adiabatic change pV^γ = constant.

After differentiation

$$p\gamma V^{\gamma-1}dV + V^\gamma dp = 0$$

or, $\gamma p = \frac{-dpV}{dV}$

$$= -\frac{dp}{dV/V} = E_{ad}$$

∴ $\frac{E_{ad}}{E_{iso}} = \gamma$ or $E_{ad} = \gamma E_{iso}$. —— (1)

Thus, the adiabatic elasticity of a gas is γ-times its isothermal elasticity.

Now $\gamma = \frac{C_p}{C_V}$, where C_p is the specific heat of a gas at constant pressure and C_V at constant volume.

$$\therefore \frac{E_{ad}}{E_{iso}} = \frac{C_p}{C_V}$$

i.e., the ratio of adiabatic to isothermal Elasticity is equal to the ratio of two specific heat.