The L-R-C Series Circuit.

Q. Discuss analytically a series resonant L-C-R Circuit.

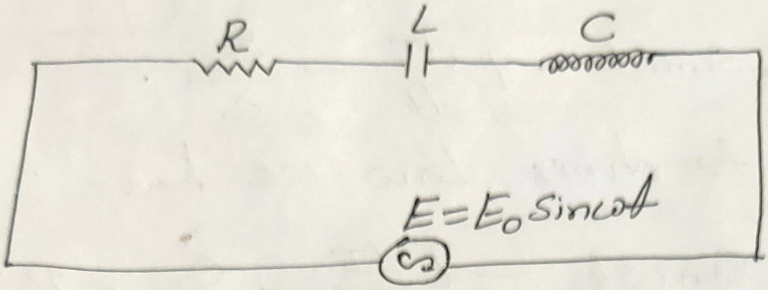

Fig 1.

A circuit containing inducter, resister & capaciter of inductance 'L' resistance 'R' and capacitance 'c' respectively all in series with a source of senusoidal emf

$E = E_0 \sin\omega t$. as shown in fig 1

It sends an alternating current through it.

If I = instantaneus value of current

& Q = the instantaneus charge on the plates of condenser. then

potential difference developed across the plates of condenser is $\frac{Q}{C}$ & induced emf due to self induction on the inducter is $L \cdot \frac{dI}{dt}$.

Both these two act in opposition to the applied emf.

$\therefore$ effective emf in the circuit at any instant is

$$E_0 \sin\omega t - L\frac{dI}{dt} - \frac{Q}{C}.$$

According to ohm's law we have:-

$$E_0 \sin\omega t - L\frac{dI}{dt} - \frac{Q}{C} = RI$$

$$\Rightarrow L\frac{dI}{dt} + RI + \frac{Q}{C} = E_0 \sin\omega t$$

After putting $I = \frac{dQ}{dt}$

$$\Rightarrow L\cdot\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E_0 \sin\omega t$$

Solution of this equn is of the form

$$Q = A\cos(\omega t + \phi)$$

where A & ϕ are constant to be determined.

Hence $$\frac{dQ}{dt} = -A\omega\sin(\omega t + \phi)$$

$$\frac{d^2Q}{dt^2} = -A\omega^2\cos(\omega t + \phi)$$

putting above value we have.

$$-\omega^2 LA\cos(\omega t + \phi) - R\omega A\sin(\omega t + \phi) + \frac{A}{C}\cos(\omega t + \phi) = E_0 \sin\omega t.$$

$$\Rightarrow A\left[\frac{1}{C}-\omega^2 L\right]\cos(\omega t+\phi)-R\omega A\sin(\omega t+\phi)$$

$$=E_0\sin\omega t$$

$$\Rightarrow A\left[\frac{1}{C}-\omega^2 L\right](\cos\omega t\cos\phi-\sin\omega t\sin\phi)-\omega AR.$$

$$(\sin\omega t\cos\phi+\cos\omega t\sin\phi)=E_0\sin\omega t \quad (1)$$

(a) At $\omega t=0$, $\sin\omega t=0$, $\cos\omega t=1$

$$\Rightarrow A\left(\frac{1}{C}-\omega^2 L\right)\cos\phi-A\omega R\sin\phi=0$$

$$\Rightarrow \tan\phi=\frac{\frac{1}{C}-\omega^2 L}{\omega R}$$

$$=\frac{\frac{1}{\omega C}-\omega L}{R}$$

$$\Rightarrow \sin\phi=\frac{\frac{1}{\omega C}-\omega L}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}$$

& $$\cos\phi=\frac{R}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}$$

(b) When $\omega t=\pi/2$, $\cos\omega t=0$ & $\sin\omega t=1$ then from equ^n (1) we have