

Work done by the engine per cycle

During the above cycle, the working substance absorbs an amount of heat Q_1 from the source and rejects Q_2 to the sink.

Hence, the net amount of heat absorbed by the gas per cycle = $Q_1 - Q_2$.

The net work done by the engine per cycle

$$= W_1 + W_2 + W_3 + W_4$$

$$= W_1 + W_3 \quad (\because W_2 = -W_4)$$

From the graph, the net work done per cycle

$$= \text{area } ABG\text{EA} + \text{area } BCHG\text{D} - \text{area } CHFDC - \text{area } DFEAD.$$

$$= \text{area } ABCDA$$

————— (5)

Thus, the area enclosed by the Carnot cycle consisting of two isothermals and two adiabatics gives the net amount of work done per cycle.

In cyclic process,
Net Heat absorbed = Net Work done per cycle.

$$\Rightarrow Q_1 - Q_2 = W_1 + W_3$$

$$= RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4}$$

Since the points A & D lie on same adiabat

$$DA, \quad \therefore T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\therefore T_2 = \frac{T_1 V_1^{\gamma-1}}{V_4^{\gamma-1}}$$

————— (7)

Similarly, points B and C lie on the same adiabatic BC.

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \quad \text{--- (8)}$$

from eqn (7) & (8)

$$\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

or $\frac{V_1}{V_4} = \frac{V_2}{V_3}$

or $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ putting in eqn (6) we get

$$\begin{aligned} \text{Net work done} &= Q_1 - Q_2 \\ &= RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_2}{V_1} \end{aligned}$$

$$\begin{aligned} \Rightarrow \omega &= (Q_1 - Q_2) \\ &= R(T_1 - T_2) \log_e \frac{V_2}{V_1} \quad \text{--- (9)} \end{aligned}$$

