B.Sc. (H) – III        Paper – V

# Curvilinear Coordinates

Point P in 3D space (see Fig) is defined by the function

$P \equiv P(u_1, u_2, u_3)$

$u_1, u_2, u_3 \rightarrow$ Single valued functions

$\left.\begin{array}{l} u_1 = \text{const} \\ u_2 = \text{const.} \\ u_3 = \text{const.} \end{array}\right\} \rightarrow$ defines coordinates surfaces.

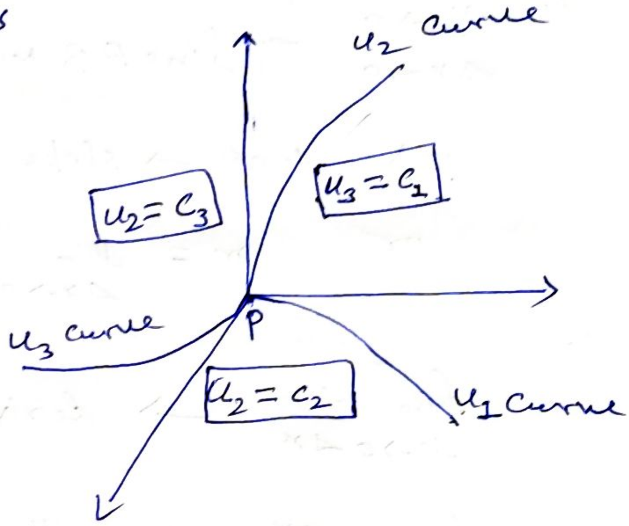


Fig.

## General Curvilinear Coordinates :-

If the orientation of the coordinates surfaces changes from point to point then $u_1$, $u_2$, and $u_3$ (or $u_i$ | $i = 1, 2, 3$) are called curvilinear coordinates.

## Orthogonal Curvilinear coordinates :-

If the three surfaces are everywhere mutually perpendicular, then $u_i$ are called orthogonal curvilinear coordinates.

Let us ~~assume~~ assume the following transformation exists

$$X = x_1 = f_1(u_1, u_2, u_3)$$
$$Y = x_2 = f_2(u_1, u_2, u_3)$$
$$Z = x_3 = f_3(u_1, u_2, u_3)$$
and
$$u_1 = F_1(x_1, x_2, x_3)$$
$$u_2 = F_2(x_1, x_2, x_3)$$
$$u_3 = F_3(x_1, x_2, x_3)$$

Let $\vec{r} = \vec{r}(u_1, u_2, u_3) \longrightarrow$ position vector of point P.

Next, element of displacement is given by

$$d\vec{r} = \frac{\partial \vec{r}}{\partial u_1} du_1 + \frac{\partial \vec{r}}{\partial u_2} du_2 + \frac{\partial \vec{r}}{\partial u_3} du_3$$

$$= \sum_{i=1}^{3} \frac{\partial \vec{r}}{\partial u_i} du_i = d\vec{s} \quad \text{—— (i)}$$

$$ds^2 = d\vec{r} \cdot d\vec{r} = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial \vec{r}}{\partial u_i} \cdot \frac{\partial \vec{r}}{\partial u_j} du_i du_j$$

$$ds^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \vec{a}_i \cdot \vec{a}_j \, du_i du_j$$

$$ds^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} g_{ij} \, du_i du_j \quad \text{—— (2)}$$

where, $g_{ij} = \vec{a}_i \cdot \vec{a}_j$ = metric coefficients.

$\vec{a}_i \cdot \vec{a}_j = \vec{a}_j \cdot \vec{a}_i \Rightarrow g_{ij} = g_{ji} \longrightarrow g_{ij}$ is symmetric