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GIRIDHAR KUMAR

[05 Feb-2024.]

MJC PHYSICS (SEM-II).

⇒ Addition of two S.H.Ms :—

Let two linear S.H.M's are

$$x_1 = A_1 \sin(\omega t + a_1) \quad \text{——— (1)}$$

$$x_2 = A_2 \sin(\omega t + a_2) \quad \text{——— (2)}$$

Where A_1, A_2 are amplitudes and a_1, a_2 are initial phase angles and x_1, x_2 are the displacement of two S.H.Ms in time 't'. and ω is same for both S.H.M's

Now the resultant displacement of two S.H.M's is

$$x = x_1 + x_2 \quad \text{——— (3)}$$

Now using Equ (1) and (2) We get

$$x_1 + x_2 = A_1 \sin(\omega t + a_1) + A_2 \sin(\omega t + a_2)$$

$$\Rightarrow \quad x = A_1[\sin \omega t \cos a_1 + \cos \omega t \cdot \sin a_1] + A_2[\sin \omega t \cdot \cos a_2 + \cos \omega t \cdot \sin a_2]$$

$$\Rightarrow \quad x = A_1 \cdot \sin \omega t \cdot \cos a_1 + A_1 \cdot \cos \omega t \cdot \sin a_1 + A_2 \cdot \sin \omega t \cdot \cos a_2 + A_2 \cdot \cos \omega t \cdot \sin a_2$$

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$\Rightarrow x = (A_1 \cdot \sin wt \cos a_1 + A_2 \sin wt \cdot \cos a_2) +$
$(A_1 \cos wt \cdot \sin a_1 + A_2 \cdot \cos wt \cdot \sin a_2)$

$\therefore x = \sin wt (A_1 \cos a_1 + A_2 \cdot \cos a_2) +$
$\cos wt (A_1 \sin a_1 + A_2 \sin a_2)$ — (4)

Now Let $A_1 \cdot \cos a_1 + A_2 \cos a_2 = R \cos \delta$ — (5.)

and $A_1 \cdot \sin a_1 + A_2 \cdot \sin a_2 = R \sin \delta$ — (6.)

Again using Equ (5.) and (6.) we get

$x = \sin wt \cdot R \cos \delta + \cos wt \cdot R \sin \delta$

$\therefore x = R \sin (wt + \delta)$ — (7.)

Here Equ (7.) represent linear S.H.M of amplitude R and initial phase angle δ with same period.

Now Squaring Equ (5.) and (6) and adding them we get

$(A_1 \cdot \cos a_1 + A_2 \cdot \cos a_2)^2 + (A_1 \cdot \sin a_1 + A_2 \cdot \sin a_2)^2$
$= R^2 \cos^2 \delta + R^2 \sin^2 \delta.$

$A_1^2 \cdot \cos^2 a_1 + A_2^2 \cdot \cos^2 a_2 + 2 A_1 \cdot A_2 \cdot \cos a_1 \cdot \cos a_2$
$+ A_1^2 \sin^2 a_1 + A_2^2 \cdot \sin^2 a_2 + 2 A_1 \cdot A_2 \cdot \sin a_1 \cdot \sin a_2$
$= R^2 (\cos^2 \delta + \sin^2 \delta)$

Using $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\Rightarrow (A_1^2 \cdot \cos^2 a_1 + A_1^2 \sin^2 a_1) + (A_2^2 \cdot \cos^2 a_2 + A_2^2 \cdot \sin^2 a_2)$
$+ 2 A_1 \cdot A_2 \cdot \cos a_1 \cdot \cos a_2 + 2 A_1 \cdot A_2 \cdot \sin a_1 \cdot \sin a_2$
$= R^2$

$\Rightarrow A_1^2 + A_2^2 + 2 A_1 \cdot A_2 (\cos a_1 \cdot \cos a_2 + \sin a_1 \cdot \sin a_2) = R^2$

Using $\boxed{\cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A-B)}$

$\Rightarrow$

$\Rightarrow A_1^2 + A_2^2 + 2 A_1 \cdot A_2 \cos(a_1 - a_2) = R^2$

$\Rightarrow R = \pm \sqrt{A_1^2 + A_2^2 + 2 A_1 \cdot A_2 \cos(a_1 - a_2)}$ — (8)

Equ (8) represent resultant amplitude of two S.H.M's

Again Dividing Equation (6) by (5) we get

$$\frac{A_1 \sin a_1 + A_2 \sin a_2}{A_1 \cos a_1 + A_2 \cos a_2} = \frac{R \sin \delta}{R \cos \delta}$$

$$\Rightarrow \frac{A_1 \sin a_1 + A_2 \sin a_2}{A_1 \cos a_1 + A_2 \cos a_2} = \tan \delta.$$

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$$\Rightarrow \delta = \tan^{-1}\left[\frac{A_1 \sin a_1 + A_2 \sin a_2}{A \cos a_1 + A_2 \cos a_2}\right] \quad \text{—— (9.)}$$

Here Eqn (9.) represent resultant or intial phase of two S.H.M's.