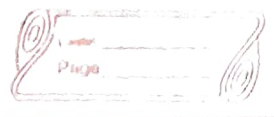


05.02.2024



Q. solve  $(D^2 + a^2)y = \sec ax$ .

Solving Questions of such type with RHS sec ax, tan ax, cosec ax are very important. Students must practice such questions many times.

For CF,  $D^2 + a^2 = 0 \Rightarrow D = \pm ai$

$\therefore$  CF =  $C_1 \cos ax + C_2 \sin ax$ .

For PI

$PI = \frac{1}{D^2 + a^2} \sec ax$

$\Rightarrow PI = \frac{1}{(D+ai)(D-ai)} \sec ax$

Now we write  $\sec ax = \frac{e^{aix} e^{-aix}}{\cos ax}$

$\Rightarrow PI = \frac{(D+ai) - (D-ai)}{2ai (D+ai)(D-ai)} \sec ax$

$\Rightarrow PI = \frac{1}{2ai} \left[ \frac{1}{D-ai} - \frac{1}{D+ai} \right] \sec ax$

$\Rightarrow PI = \frac{1}{2ai} \left[ \frac{e^{aix} \cdot e^{-aix}}{\cos ax} - \frac{e^{-aix} \cdot e^{aix}}{\cos ax} \right]$

[Very important step]

$$\therefore PI = \frac{1}{2a_i(D-a_i)} e^{aix} \cdot \frac{e^{-aix}}{\cos ax}$$

$$= \frac{1}{2a_i(D+a_i)} e^{-aix} \cdot \frac{e^{aix}}{\cos ax} \quad \text{--- (A)}$$

Now 1st part of eq. (A)

$$PI \text{ of 1st part} = \frac{1}{2a_i(D-a_i)} e^{aix} \cdot \frac{e^{-aix}}{\cos ax}$$

Here we will integrate  $e^{aix}$ .

For this, we shall replace  $D$  by  $D+a_i$  (where  $a_i$  is  $aix$ )

$$= \frac{e^{aix}}{2a_i} \cdot \frac{1}{(D+a_i)-a_i} \cdot \frac{e^{-aix}}{\cos ax}$$

$$= \frac{e^{aix}}{2a_i} \cdot \frac{1}{D} \cdot \frac{e^{-aix}}{\cos ax}$$

$$= \frac{e^{aix}}{2a_i} \int \frac{e^{-aix}}{\cos ax} dx$$

of 1st part

$$\Rightarrow PI = \frac{e^{aix}}{2ai} \int \frac{\cos ax - i \sin ax}{\cos ax} dx$$

of 1st part

$$\Rightarrow PI = \frac{e^{aix}}{2ai} \int [1 - i \tan ax] dx$$

of 1st part

$$\Rightarrow PI = \frac{e^{aix}}{2ai} \left[ x + \frac{i}{a} \log \cos ax \right] \quad (B)$$

Again

PI of 2nd part

$$= \frac{1}{2ai(D+a)} e^{-aix} \cdot \frac{e^{aix}}{\cos ax}$$

$$= \frac{e^{-aix}}{2ai} \frac{1}{[(D-ai)+ai]} \frac{e^{aix}}{\cos ax}$$

$$= \frac{e^{-aix}}{2ai} \frac{1}{D} \frac{e^{aix}}{\cos ax} = \frac{e^{-aix}}{2ai} \int \frac{e^{aix}}{\cos ax} dx$$

$$= \frac{e^{-aix}}{2ai} \int \frac{\cos ax + i \sin ax}{\cos ax} dx$$

$$= \frac{e^{-aix}}{2ai} \int [1 + i \tan ax] dx$$

∴ PI of 2nd part

$$= \frac{e^{-aix}}{2ai} \left[ x + \frac{i}{a} \log \sec ax \right]$$

$$= \frac{e^{-aix}}{2ai} \left[ x - \frac{i}{a} \log \cos ax \right] \quad \text{--- (B)}$$

using eq (B) and eq (C) in (A)

we have

$$PI = \frac{e^{aix}}{2ai} \left[ x + \frac{i}{a} \log \cos ax \right]$$

$$- \frac{e^{-aix}}{2ai} \left[ x - \frac{i}{a} \log \cos ax \right]$$

$$\Rightarrow PI = \frac{x}{2ai} \left[ e^{aix} - e^{-aix} \right]$$

$$+ \frac{\log \cos ax}{2a^2} \left[ e^{aix} + e^{-aix} \right]$$

$$\Rightarrow PI = \frac{x}{2ai} \times 2i \sin ax$$

$$+ \frac{\log \cos ax}{2a^2} \times 2 \cos ax$$

$$\Rightarrow PI = \frac{x}{a} \sin ax + \frac{\cos ax}{a^2} \log \cos ax.$$

Hence the complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = C_1 \cos ax + C_2 \sin ax + \frac{x}{a} \sin ax + \frac{\cos ax}{a^2} \log \cos ax.$$