

The total energy of an electron in a particular orbit is calculated by adding its potential energy & kinetic energy.

$$E_{\text{Total}} = P.E. + K.E.$$

The kinetic energy of the electron = $\frac{1}{2} m u^2$

Potential energy = $-\frac{k Z e^2}{r}$

Hence Total Energy = $P.E. + K.E.$

$$= -\frac{k Z e^2}{r} + \frac{1}{2} m u^2 \quad \text{--- (1)}$$

We know that the centrifugal force is equal to Coulombic attraction force

So, $\frac{m u^2}{r} = \frac{k Z e^2}{r^2}$

or $m u^2 = k \cdot \frac{Z e^2}{r} \quad \text{--- (2)}$

On substituting the value of $m u^2 = \frac{k Z e^2}{r}$ in equation (1)

we have,

$$E_{\text{Total}} = -\frac{k Z e^2}{r} + \frac{k \cdot Z e^2}{2r}$$

or $E_{\text{Total}} = \frac{k Z e^2}{2r} - \frac{k Z e^2}{r} = \frac{k \cdot Z e^2}{r} \left(\frac{1}{2} - 1 \right)$

$$= \frac{k \cdot Z e^2}{r} \left(\frac{1-2}{2} \right)$$

$$= -\frac{k \cdot Z e^2}{2r} \quad \text{--- (3)}$$

Again substituting the value of

$$r = \frac{n^2 h^2}{4 \pi^2 m \cdot Z e^2 \cdot k} \quad \text{in equation (3)}$$

$$E_{\text{Total}} = -\frac{k \cdot Z e^2}{2} \times \frac{4 \pi^2 \cdot m \cdot Z e^2 \cdot k}{n^2 h^2}$$

$$= -\frac{2 \pi^2 Z^2 e^4 \cdot m \cdot k^2}{n^2 h^2} \quad \text{--- (14)}$$

Thus the total energy of electron in an orbit of hydrogen atom is given by

$$E_n = -\frac{2 \pi^2 Z^2 e^4 m k^2}{n^2 h^2}$$

where E_n is the total energy of electron in n^{th} orbit.