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MJC, SEM-II, Physics, Notes

12/04/24

Topic :- Vibration of string :-

A vibration in a string is a wave. Resonance causes a vibrating to produce a sound with constant frequency, i.e. constant pitch. If the length or tension of the string is correctly adjusted, the sound produced is a musical tone.

Vibrating strings are the basis of string instruments such as guitars, cellos and pianos.

\* The velocity of propagation of a wave in a string ( $v$ ) is proportional to the square root of the force of tension of the string ( $T$ ) and inversely proportional to the square root of the linear density ( $\mu$ ) of the string.

$$v = \sqrt{\frac{T}{\mu}}$$

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### Derivation:-

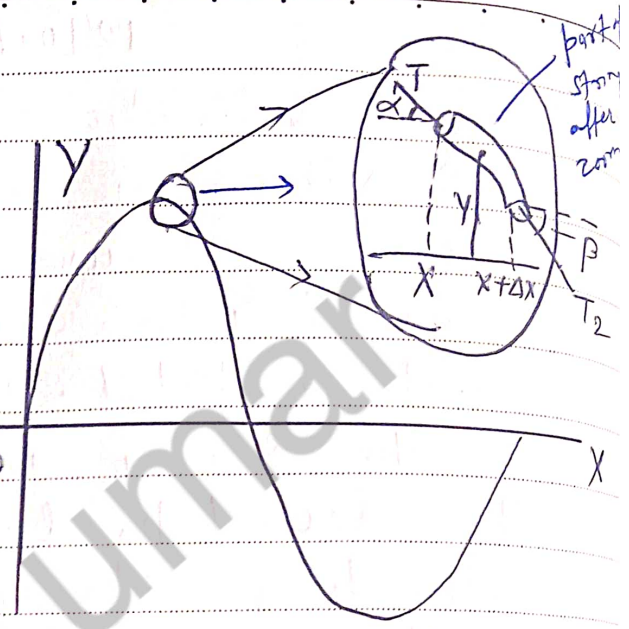
Let  $\Delta x$  be the length of a piece of string,  $m$  its mass and  $\mu$  is its linear density. If angle  $\alpha$  and  $\beta$  are small,

then the horizontal components of tension on either side can both be approximated by a constant  $T$ , for which the net horizontal force is zero. By using the small angle approximation the horizontal tension acting on both sides of the string segment are given by

$$T_{1x} = T_1 \cos(\alpha) \approx T$$

$$T_{2x} = T_2 \cos(\beta) \approx T$$

From Newton's second law for the vertical component, the mass of the piece



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times its acceleration,  $a$ , will be equal to the net force on the piece.

$$\begin{aligned}\sum F_y &= T_{1y} - T_{2y} = -T_2 \sin(\beta) + T_1 \sin(\alpha) \\ &= \Delta m a \approx \mu \Delta x \frac{\partial^2 y}{\partial t^2}\end{aligned}$$

Dividing this expression by  $T$  and substituting the first and second equation

$$\begin{aligned}-\frac{T_2 \sin(\beta)}{T_2 \cos(\beta)} + \frac{T_1 \sin(\alpha)}{T_1 \cos(\alpha)} &= -\tan \beta + \tan \alpha \\ &= \frac{\mu \Delta x}{T} \frac{\partial^2 y}{\partial t^2}\end{aligned}$$

According to the small-angle approximation,

$$\frac{1}{\Delta x} \left( \frac{\partial y}{\partial x} \Big|^{x+\Delta x} - \frac{\partial y}{\partial x} \Big|^{x} \right) = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2}$$

In the limit that  $\Delta x$  approaches zero,

$$\therefore \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2}$$

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$$\Rightarrow \frac{\partial x^2}{\partial t^2} = \frac{T}{\mu}$$

$$\left[ \therefore \frac{\partial x^2}{\partial t^2} = v^2 \right]$$

$$\Rightarrow v^2 = \frac{T}{\mu}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{T}{\mu}}}$$

Proved,