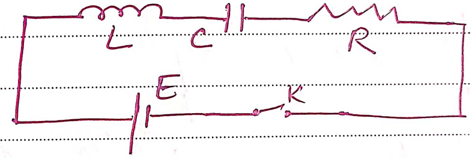
> Date ☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun

Degree - II, paper - IV, Group - B, 15/02/2024

Name :- Giridhar Kumar

- Topic :- Growth of charge - LCR.

In this circuit, when 'K' switch is pressed, the capacitor is charged. Let 'Q' be the charge on capacitor and I is the current in the circuit at an ~~int~~ instant 't' during charging.



- potential difference across the capacitor $= \frac{Q}{C}$

- Potential difference across the inductance,

$$= L\frac{dI}{dt}$$

- Potential difference across the resistor

$$= IR$$

Adding up all the potential difference

$$\boxed{L\frac{dI}{dt} + RI + \frac{Q}{C} = E} \longleftarrow (1)$$

$$\because I = \frac{dQ}{dt} \rightarrow L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

Mon Tue Wed Thu Fri Sat Sun

Degree – II, paper – IV, Group-B, 15/02/2024

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$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q - CE}{LC} = 0$$

Let us subsitute $\frac{R}{L} = 2b$ and $\frac{1}{LC} = k^2$

$$\boxed{\frac{d^2 q}{dt^2} + 2b\frac{dq}{dt} + k^2(q - CE) = 0} \quad \text{--- (2)}$$

Let $x = q - CE$, then,

$$\boxed{\frac{d^2 x}{dt^2} + 2b\frac{dx}{dt} + k^2 x = 0} \quad \text{--- (3)}$$

Hence, the most general solution of eqn (3)

$$\boxed{x = Ae^{(-b+\sqrt{b^2-k^2})t} + Be^{(-b-\sqrt{b^2-k^2})t}}$$

We can consider $CE = q_0$ = Final steady charge on the capacitor

$\therefore \quad x = q - CE = q - q_0$

$$\therefore \quad q - q_0 = Ae^{(-b+\sqrt{b^2-k^2})t} + Be^{(-b-\sqrt{b^2-k^2})t}$$

$$\Rightarrow \boxed{q = q_0 + Ae^{(-b+\sqrt{b^2-k^2})t} + Be^{(-b-\sqrt{b^2-k^2})t}} \quad \text{--- (4)}$$

> Date

☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun

Degree-II, paper-IV, Group-B, 15/07/2024

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using initial condition;

at, $t=0$, $\phi=0$

$$\therefore \quad 0=\phi_0+(A+B)$$

or $\boxed{A+B=-\phi_0}$ —— (5)

By differentiating equ-(4) w.r.t time(t)

$$\frac{d\phi}{dt}=0+A\left(-b+\sqrt{b^2-k^2}\right)e^{(-b+\sqrt{b^2-k^2})t}+B\left(-b-\sqrt{b^2-k^2}\right)e^{(-b-\sqrt{b^2-k^2})t}$$

at $t=0$, $\frac{d\phi}{dt}=0$

$$0=A\left(-b+\sqrt{b^2-k^2}\right)+B\left(-b-\sqrt{b^2-k^2}\right)$$

$$0=-bA+A\sqrt{b^2-k^2})-bB-B\sqrt{b^2-k^2}$$

$$b(A+B)=\sqrt{b^2-k^2}\,(A-B)$$

from Eqn (5.)

$$-\frac{\phi_0 b}{\sqrt{b^2-k^2}}=A-B \quad \text{—— (6.)}$$

Adding Eqn (5) and (6.)

$$2A=\frac{-\phi_0(b+\sqrt{b^2-k^2})}{\sqrt{b^2-k^2}}$$

> Date ☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun

Degree - II, paper - IV, Group - B 15/07/2024

Giridhar Kum

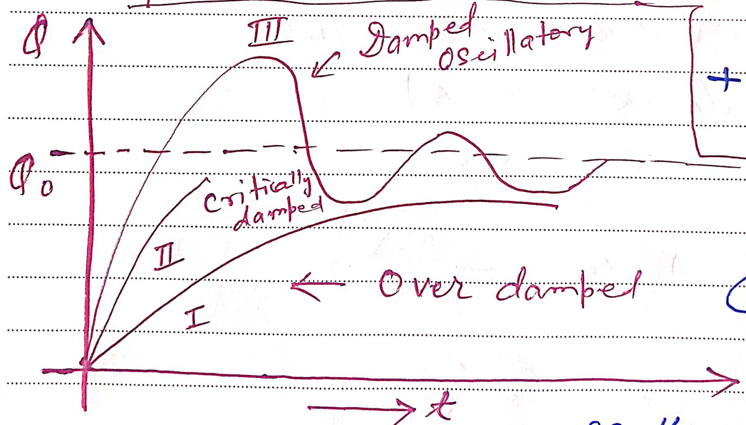
$$\Rightarrow A = -\cancel{\frac{1}{2}Q_0}\, \frac{1+b}{\sqrt{b^2-K^2}}$$

$$\Rightarrow \boxed{A = -\frac{1}{2}Q_0\left(1+\frac{b}{\sqrt{b^2-K^2}}\right)} \quad \text{--- (7)}$$

$$\boxed{B = -\frac{1}{2}Q_0\left(1-\frac{b}{\sqrt{b^2-K^2}}\right)} \quad \text{--- (8)}$$

Now subsituting the value of A and B in Eqn (4) we get

$$\boxed{Q = Q_0 - \frac{1}{2}Q_0 e^{-bt}\left[\left(1+\frac{b}{\sqrt{b^2-K^2}}\right)e^{\sqrt{b^2-K^2}\,t} + \left(1-\frac{b}{\sqrt{b^2-K^2}}\right)e^{-\sqrt{b^2-K^2}\,t}\right]} \quad \text{--- (9)}$$



Case I:

If $b^2 > K^2 \rightarrow \sqrt{b^2-K^2}$ is real. The charge on the capacitor grows exponentially with time and attains the maximum value Q_0 asymptotically. The charge is known as "Over dead beat".

Date

☐ Mon ☐ Tue ☐ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun

Degree - II paper - IV Group - B, 15/07/2024

Case II.

If $b^2 = k^2$, the charge rises to the maximum value q_0 in a short time. Such as charge is called critically damped.

Case III

If $b^2 < k^2 \rightarrow \sqrt{b^2 - k^2}$ is imaginary

Let $\sqrt{b^2 - k^2} = i\omega$ where $i = \sqrt{-1}$ and

$\omega = \sqrt{k^2 - b^2}$

Then equ (9) can be written as;

$$q = q_0 - \frac{1}{2} q_0 e^{-bt}\left[\left(1 + \frac{b}{i\omega}\right)e^{i\omega t} + \left(1 - \frac{b}{i\omega}\right)e^{-i\omega t}\right]$$

$$q = q_0 - q_0 e^{-bt}\left[\frac{e^{i\omega t} + e^{-i\omega t}}{2} + \frac{b}{\omega}\left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right)\right]$$

$$q = q_0 - q_0 e^{-bt}\left[\cos\omega t + \frac{b}{\omega}\sin\omega t\right]$$

$$q = q_0\left[1 - \frac{e^{-bt}}{\omega}(\omega\cos\omega t + b\sin\omega t)\right]$$

Let $\omega = k\sin\alpha$ and $b = k\cos\alpha$

$\therefore \tan\alpha = \frac{\omega}{b}$

$$q = q_0\left[1 - \frac{e^{-bt}}{\omega}(k\sin\alpha\cos\omega t + k\cos\alpha\sin\omega t)\right]$$

Degree-II, paper-IV, Group-B, 15/07/2024

$$q = q_0\left[1 \frac{Ke^{-bt}}{\omega} \sin(\omega t + K)\right] \quad \text{—— (10)}$$

In the begin, we considered,

$$K = \sqrt{\frac{1}{LC}} \quad \text{and} \quad b = \frac{R}{2L}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$q = q_0\left[1 - \frac{e^{-\frac{R}{2L}t}\sqrt{1/LC}}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\, t + \alpha\right)\right]$$

This equ represents a damped oscillatory charge as shown by the caurve III. The charge oscillates above and below q_0 till it finally settles down to q_0 value. The frequency of oscillation in the circuit is given by

$$f = \omega/2\pi = \frac{\sqrt{K^2 - b^2}}{2\pi} = \frac{1}{2\pi}\sqrt{1/LC - R^2/4L^2}$$

Where $R = 0 \rightarrow f = \frac{1}{2\pi\sqrt{LC}}$