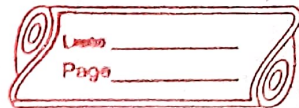


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Vector Calculus

Q. Imp. Sum

If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $\text{curl}(\text{grad}\phi)$.

Soln.

$$\text{Grad } \phi = \nabla \phi$$

$$= \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \phi$$

$$\Rightarrow \text{Grad } \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \quad (1)$$

Now $\phi = x^3 + y^3 + z^3 - 3xyz$

$$\Rightarrow \left. \begin{aligned} \frac{\partial \phi}{\partial x} &= 3x^2 - 3yz, & \frac{\partial \phi}{\partial y} &= 3y^2 - 3zx \\ \frac{\partial \phi}{\partial z} &= 3z^2 - 3xy \end{aligned} \right\} (2)$$

∴, putting the values from (2) in (1), we get

$$\therefore \text{grad } \phi = \vec{i} (3x^2 - 3yz) + \vec{j} (3y^2 - 3zx) + \vec{k} (3z^2 - 3xy)$$

$$\Rightarrow \text{grad } \phi = 3 \left[(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k} \right]$$

(3)

Now, curl grad ϕ

$$= \nabla \times \text{grad } \phi$$

$$\Rightarrow \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \left[(x^2 - yz) \vec{i} + (y^2 - zx) \vec{j} + (z^2 - xy) \vec{k} \right]$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 - yz & y^2 - zx & z^2 - xy \end{vmatrix}$$

$$= \vec{i} \left\{ \frac{\partial}{\partial y} (z^2 - xy) - \frac{\partial}{\partial z} (y^2 - zx) \right\}$$

$$- \vec{j} \left\{ \frac{\partial}{\partial x} (z^2 - xy) - \frac{\partial}{\partial z} (x^2 - yz) \right\}$$

$$+ \vec{k} \left\{ \frac{\partial}{\partial x} (y^2 - zx) - \frac{\partial}{\partial y} (x^2 - yz) \right\}$$

$$= \vec{i} \{-x + x\} - \vec{j} \{-x + x\} + \vec{k} \{-x + x\}$$

$$= \cancel{0} \cdot 3 \times \vec{0} = \vec{0}$$

Ans