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Vector calculus

Q $\boxed{\text{Impt. sum}}$

If $\phi = x^3+y^3+z^3-3xyz$, find curl(grad ϕ).

Soln. →

$$\text{Grad}\,\phi = \nabla\phi$$

$$= \left(\vec{i}\frac{\partial}{\partial x}+\vec{j}\frac{\partial}{\partial y}+\vec{k}\frac{\partial}{\partial z}\right)\phi$$

$$\Rightarrow \text{Grad}\,\phi = \vec{i}\frac{\partial\phi}{\partial x}+\vec{j}\frac{\partial\phi}{\partial y}+\vec{k}\frac{\partial\phi}{\partial z} \quad -(1)$$

Now $\phi = x^3+y^3+z^3-3xyz$

$$\left.\begin{aligned}&\Rightarrow \frac{\partial\phi}{\partial x} = 3x^2-3yz,\quad \frac{\partial\phi}{\partial y} = 3y^2-3zx\\ &\frac{\partial\phi}{\partial z} = 3z^2-3xy\end{aligned}\right] \quad -(2)$$

So, putting the values from (2) in (1), we get

$$\therefore \text{grad}\,\phi = \vec{i}(3x^2-3yz)+\vec{j}(3y^2-3zx)$$
$$+\vec{k}(3z^2-3xy)$$

$$\Rightarrow \text{grad}\,\phi = 3\left[(x^2-yz)\vec{i}+(y^2-zx)\vec{j}+(z^2-xy)\vec{k}\right] \quad -(3)$$

Now, curl grad ϕ

$$= \nabla \times \text{grad}\,\phi$$

$$= 3\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}\right]$$

$$= 3\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 - yz & y^2 - zx & z^2 - xy \end{vmatrix}$$

$$= 3\left[\vec{i}\left\{\frac{\partial}{\partial y}(z^2 - xy) - \frac{\partial}{\partial z}(y^2 - zx)\right\} - \vec{j}\left\{\frac{\partial}{\partial x}(z^2 - xy) - \frac{\partial}{\partial z}(x^2 - yz)\right\} + \vec{k}\left\{\frac{\partial}{\partial x}(y^2 - zx) - \frac{\partial}{\partial y}(x^2 - yz)\right\}\right]$$

$$= 3\left[\vec{i}\{\cancel{-x} + \cancel{x}\} - \vec{j}\{\cancel{-y} + \cancel{y}\} + \vec{k}\{\cancel{-z} + \cancel{z}\}\right]$$

$$= 3 \times \vec{0} = \vec{0}$$

Ans