Diff. Equations

Exact equations

① condition for a differential equation being exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

if the eqn is $M\,dx + N\,dy = 0$.

Soln →

$\int [M\,dx]$ treating y as a constant

$+ \int [$ those terms of N which are independent of x $]\,dy = 0$

Q. Prove that the diff. eqn.

$(y^4 + 4x^3y + 3x)\,dx + (x^4 + 4x^3y^3 + y + 1)\,dy = 0$

is exact. and find the solution

Sol. Here

$M = y^4 + 4x^3y + 3x$

$N = x^4 + 4xy^3 + y + 1$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y^4 + 4x^3y + 3x)$$

$$\Rightarrow \frac{\partial M}{\partial y} = 4y^3 + 4x^3 \quad \text{---(1)}$$

Also, $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^4 + 4xy^3 + y + 1)$

$$\Rightarrow \frac{\partial N}{\partial x} = 4x^3 + 4y^3 \quad \text{---(2)}$$

From (1) and (2), $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow$ Differential ~~equation~~ is exact.

Its soln is

$$\int [M\,dx] \text{ treating } y \text{ as constant} + \int [\text{those terms of } N \text{ which do not have } x]\,dy = 0$$

$$\Rightarrow \int (y^4 + 4x^3y + 3x)\,dx + \int (y+1)\,dy = 0$$

$$\Rightarrow y^4 \int dx + 4y \int x^3\,dx + 3\int x\,dx + \frac{y^2}{2} + y = K$$

$$\Rightarrow xy^4 + x^4y + \frac{3}{2}x^2 + \frac{y^2}{2} + y = K$$