Degree-III, Paper-V, Group-C.

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Topic:- Schrodinger wave Equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \psi + V(x)\psi . \quad \text{—— (I)}$$

This equation does, in fact, satisfy Ehrenfest's principle.

$$\therefore \langle p \rangle = m \partial_t \langle x \rangle$$

$$= m \int dx \left\{ \frac{\partial \psi^*}{\partial t} x \psi + \psi^* x \frac{\partial \psi}{\partial t} \right\} \quad \text{—— (II)}$$

Then rewriting the Schrodinger equation as,

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} - \frac{i}{\hbar} V\psi .$$

$$\frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m} \cdot \frac{\partial^2 \psi^*}{\partial x^2} + \frac{i}{\hbar} V\psi^* \quad \text{—— (III)}$$

Now putting value in Equ (II)

$$\langle p \rangle = m \int dx \left\{ \left(-\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + \frac{i}{\hbar} V\psi^* \right) x\psi \right.$$

$$= \int dx\, \psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) \psi \quad \text{—— (IV)}$$

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above equation is the same rule for computing $\langle P \rangle$ as in the previous $V=0$ case.

Then

$$\partial t \langle P \rangle = -i\hbar \int dx \left[\frac{\partial \psi^*}{\partial t} \cdot \frac{\partial \psi}{\partial x} + \psi^* \frac{\partial^2 \psi}{\partial t \partial x} \right]$$

$$= i\hbar \int dx \left[\frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \cdot \frac{\partial \psi}{\partial t} \right]$$

$$= \cancel{i\hbar}$$

$$= \cancel{-i\hbar \int dx \left\{ \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} \right.}$$

$$= -i\hbar \int dx \left\{ \left(-\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + \frac{i}{\hbar} V\psi \right) \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \left(\frac{i\hbar}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} - \frac{i}{\hbar} V\psi \right) \right\}$$

$$= -\frac{\hbar^2}{2m} \int dx \left[\frac{\partial^2 \psi^*}{\partial x^2} \cdot \frac{\partial \psi}{\partial x} + \frac{\partial \psi^*}{\partial x} \cdot \frac{\partial^2 \psi}{\partial x^2} \right] + \int dx \left[\psi^* V \frac{\partial \psi}{\partial x} + \frac{\partial \psi^*}{\partial x} V\psi \right] \quad \text{—— (V)}$$

Again applying integration by parts to the first term of the first integral, and the second term of the second integral, we find that

$$\partial t \langle P \rangle = -\frac{\hbar^2}{2m} \int dx \left[-\frac{\partial \psi^*}{\partial x} \cdot \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi^*}{\partial x} \frac{\partial^2}{\partial x^2} \right.$$

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$$+\int dx\left[\psi^* V\frac{\partial\psi}{\partial x}-\psi^*\frac{\partial}{\partial x}(V\psi)\right]$$

$$=\int dx\left[\psi^* V\frac{\partial\psi}{\partial x}-\psi^* V\frac{\partial\psi}{\partial x}-\psi^*\frac{\partial V}{\partial x}\psi\right]$$

$$=\int dx\,\psi^*\left(-\frac{\partial V}{\partial x}\right)\psi$$

$$=\left\langle -\frac{\partial V}{\partial x}\right\rangle \quad \text{—— (VI)}$$

exactly as required by Ehrenfeast's principle.

Now $\langle p\rangle = \int dx\,\psi^*(x,t)\,\tilde{p}\,\psi(x,t)$ ——

Where $\tilde{p}$ is the differential operator known as the Momemntum operator.

$$\boxed{\tilde{p}\equiv -i\hbar\frac{\partial}{\partial x}} \quad \text{—— (VII)}$$

Above Equation can be used to express any x-derivative in term of the momemntum operator and in particular

$$\boxed{\frac{\partial^2}{\partial x^2}=-\frac{1}{\hbar^2}\tilde{p}^2} \quad \text{—— (VIII)}$$

The the schrodinger eqn can be written

in the compact form

$$i\hbar\partial_t\Psi = \left(\frac{\tilde{P}^2}{2m} + V\right)\Psi$$

$$= \tilde{H}\Psi$$

where

$$\boxed{H[P, q] = \frac{P^2}{2m} + V}$$

It is just the Hamiltonian for a particle of mass m, moving in a potential field V, and $\tilde{H}$ is the Hamiltonian operator

$$\boxed{\tilde{H} = H[\tilde{P}, x]}$$