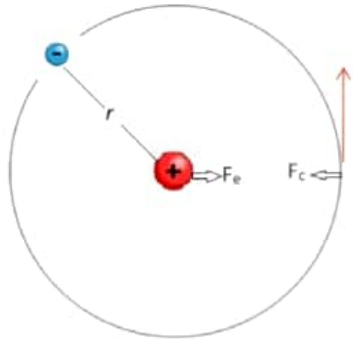
Assume a circular orbit for electrons for convenience



The centripetal force $F_c = \frac{mv^2}{r}$

This force holding the electrons in an orbit of radius r from the nucleus is provided by the centripetal force $F_e = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$

For stable orbits $F_c = F_e$ $\quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$

The electron velocity v is related to the orbit radius r by the formula

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o m r}}$$

Kinetic energy of electron in the orbit $T = \frac{1}{2}mv^2$

Potential energy of the system $V = -\frac{e^2}{4\pi\varepsilon_o r}$

The total energy E = T + V

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_o r} = -\frac{e^2}{8\pi\varepsilon_o r}$$

The total energy of the atomic electron is negative. This is necessary for a bound orbit.

If the energy is zero or positive the electron is not bound to the nucleus. Experiments show that 13.6 ev energy is needed to remove the electron from the hydrogen atom orbit.

$$r = -\frac{e^2}{8\pi\varepsilon_o E} = 5.3 \times 10^{-11} m$$

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Wave behaviour of electron in the Bohr orbit $\lambda = \frac{h}{mv}$

The electron speed v is given by $v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}} = 33 \times 10^{-11} m$

$$\lambda = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_0 r}{m}} = 33 \times 10^{-11} m$$

The circumference of the first Bohr orbit is $2\pi r = 33 \times 10^{-11} m$

The circumference is same as the wavelength of the electron. The orbit of the electron in a hydrogen atom corresponds to one complete electron wave joined on itself.

The fact that the electron orbit in a hydrogen atom is one electron wavelength in circumference provides the clue needed to construct a theory of the atom.

Using the concept of the electron matter wave, de Broglie provided a rationale for the quantization of the electron's angular momentum in the hydrogen atom, which was postulated in Bohr's quantum theory.

The physical explanation for the first Bohr quantization condition comes naturally when we assume that an electron in a hydrogen atom behaves not like a particle but like a wave.

Imagine a stretched string that is clamped at both ends and vibrates in one of its normal modes. If the length of the string is l, the wavelengths of these vibrations cannot be arbitrary but must be such that an integer k number of half-wavelengths fit exactly on the distance l between the ends.

This is the condition for a standing wave on a string.

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