1.19 TWO-BODY HARMONIC OSCILLATOR

A system of two masses (bodies) connected by a spring, so that both are free to oscillate simple harmonically along the length of the spring constitutes a "two-body harmonic oscillator" or a coupled oscillator.

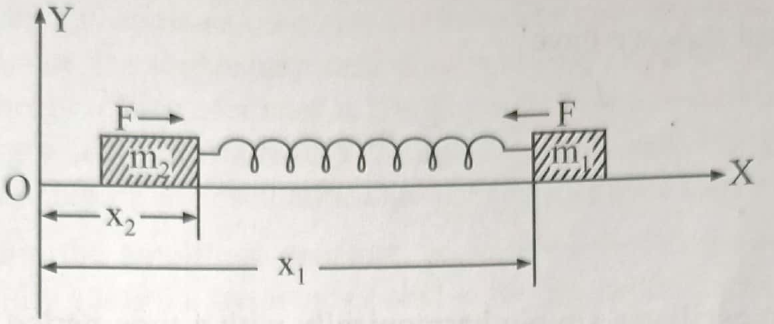

Fig. 1.11

Examples:

Diatomic molecules like H_2, CO, HCl etc. can oscillate along their respective axes of symmetry. Electro magnetic forces which couple two atoms in a di-atomic molecule, act like a tiny and massless spring.

Let us consider, the general case of two-body oscillator consisting of two masses m_1 and m_2 connected by a spring of force constant c and are free to oscillate on a frictionless surface.

Let the normal length of the spring be l and let, at any given instant, the co-ordinate of the two ends of the spring be x_1 and x_2

Extension of the spring, $x = (x_1 - x_2) - l$

Where x is positive, if the spring is stretched; x is zero, if the spring has its normal length and x is –ve, if the spring is compressed. Here, we assume x to be +ve.

The forces (F) exerted by the spring on the two masses are equal in magnitude but opposite in sign, the magnitude of each being cx.

Let $\frac{d^2x_1}{dt^2}$ be the acceleration of mass m_1 and $\frac{d^2x_2}{dt^2}$ be the acceleration of mass m_2, then

$$m_1 \frac{d^2x_1}{dt^2} = -cx \qquad \text{... }(i)$$

and

$$m_2 \frac{d^2x_2}{dt^2} = +cx \qquad \text{... }(ii)$$

Multiplying eq. (i) by m_2 and eq. (ii) by m_1, and subtracting, we get

$$m_1 m_2 \frac{d^2x_1}{dt^2} - m_1 m_2 \frac{d^2x_2}{dt^2} = -m_2 cx - m_1 cx$$

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