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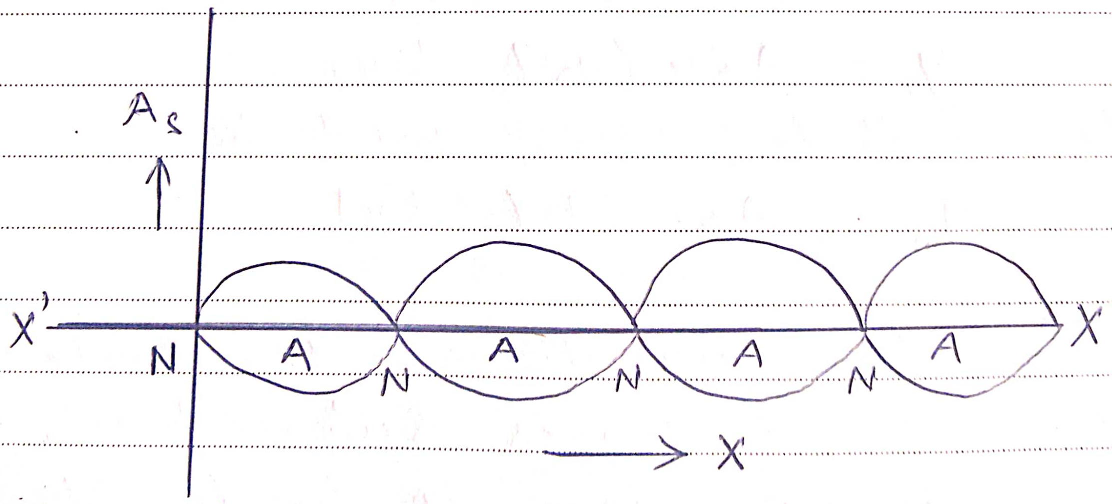
MJC, SEM-II, Notes,

Topic:- Stationary Wave as combination of Oscillations:-

When two progressive waves, having the same amplitude and period but travelling in opposite direction with the same velocity superimpose, a disturbance that does not appear to travel in space is obtained. Such a wave is called stationary wave or standing wave.



In above figure, Here waves are ~~prog~~ propagating along X-axis and width of the loop at any point represent the amplitude (A_s) of oscillation at that point. This shows that the amplitude of oscillations varies with distance along the propagation of the wave.

The points marked 'N' are called Nodes. Here the amplitude of the oscillation is zero. And points 'A' marked as antinotes. Here the amplitude of the particles is maximum

* Equation of stationary wave:—

Let the incident wave be,

$$y_1 = A\sin(\omega t - kx)$$

Its reflected wave will be

$$y_2 = A\sin(\omega t + kx)$$

If the above mentioned incident and reflected waves superimpose, a stationary wave is produced. The equation of the resultant stationary wave.

$$y = y_1 + y_2 + 2A\cos kx \sin \omega t$$

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* In general, the equations for the stationary wave may be written as:-

$$y = \pm A \cos kx \sin wt$$
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(a) The equation $y = 2A \cos kx \sin wt$ satisfied the defferential equation.

$$\frac{d^2y}{dx^2} = \frac{1}{c^2} \cdot \frac{d^2y}{dt^2}$$

Where $\boxed{c = \frac{w}{K}}$. Hence, it is equation of wave.

(b) The amplitude of the stationary wave is

$$\boxed{A_s = 2A \cos kx}$$

The amplitude of the stationary wave is not constant. It varies with x.

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The point at which the amplitude is zero is called node and point at which the amplitude is maximum is called ~~antt~~ antinode.

(c) For, node, we have $A_s = 2A\cos kx = 0$

Hence, $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{---} (2n+1)\frac{\pi}{2},$

Where $n = 0, 1, 3 \text{ ----}$

Since $k = \frac{2\pi}{\lambda}$, hence for the node we have

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \text{ --- } (2n+1)\frac{\lambda}{4}.$$

(d) The separation between the node is equal to $\frac{\lambda}{2}$.

(e) For the antinode, we have the amplitude maximum and equal to 2A.

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Hence $\cos kx = \pm 1$

So, $Kx = 0, \pi, 2\pi, 3\pi, \ldots n\pi$

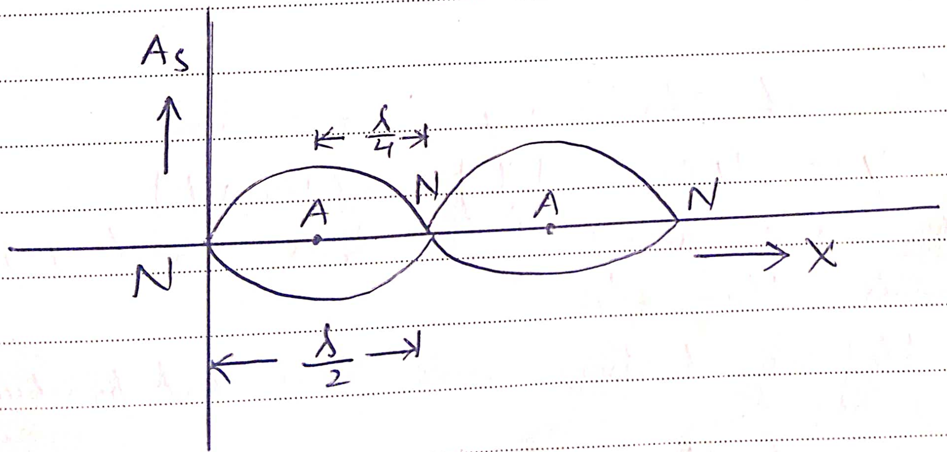
Where $n = 0, 1, 2, 3, \ldots$

$\because K = \frac{2\pi}{\lambda}$, therefore, we have

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \frac{n\lambda}{2}$

(f) The separation between the two antinodes is again $\frac{\lambda}{2}$.

(g.) The separation between nodes & consecutive antinode is $\frac{\lambda}{4}$.



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(h) The amplitude of the stationary wave varies between 0 and 2A, where A is the amplitude of the ~~st~~ superimposing incident and reflected waves.
The velocity of the particle is

$$\boxed{v = \frac{dy}{dt} = \frac{d}{dt}(2A\cos kx \sin \omega t)}$$

$$\boxed{v = 2\omega A \cos kx \cos \omega t.}$$

The maximum velocity depends on the position of the particle. It is zero for the particle at the nodes and largest for the particles at the antinodes.

* Important Question.

Q The equation of stationary wave is.

$$y = -4\sin\left(\frac{\pi x}{5}\right)\cos(100\pi t)$$

where y and x are in metres and t is in seconds then find.

(i) the amplitude of progressive wave that produce stationary wave.

(ii) the velocity of the progressive wave.

(iii) the frequency of the progressive wave