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U.G. Sem II MJC-2 (Math)

Successive Differentiation

① If $y = \sin(m \sin^{-1} x)$, prove that

$(1-x^2) y_{n+2} - (2n+1)x \cdot y_{n+1} + (m^2 - n^2) y_n = 0$

Soln:- We have $y = \sin(m \sin^{-1} x)$

$$\frac{dy}{dx} = \cos(m \sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1 = \frac{m}{\sqrt{1-x^2}} \cos(m \sin^{-1} x)$$

$$\Rightarrow \sqrt{1-x^2} \cdot y_1 = m \cos(m \sin^{-1} x)$$

$$\Rightarrow (1-x^2) y_1^2 = m^2 \cos^2(m \sin^{-1} x)$$

$$(1-x^2) y_1^2 = m^2 \cos^2(m \sin^{-1} x)$$

$$= m^2 [1 - \sin^2(m \sin^{-1} x)]$$

$$= m^2 (1 - y^2)$$

Differentiating it again, we get

$$(1-x^2) \cdot 2 y_1 y_2 - 2x y_1^2 = m^2(-2 y y_1)$$

$$\Rightarrow (1-x^2) y_2 - x y_1 + m^2 y = 0$$

Differentiating n times by Leibniz theorem, we get

$$\left[(1-x^2)y_{n+2} + n(-2x)y_{n+1} + \frac{n(n-1)}{\lfloor 2}(-2)y_n\right]$$

$$-\left[x y_{n+1} + n(1)y_n\right] + m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)x y_{n+1}$$

$$+ (m^2 - n^2 + n - n)\, y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)x y_{n+1}$$

$$+ (m^2 - n^2)\, y_n = 0$$

<u>Proved</u>.