

Differential equation of damped harmonic oscillator.

Restoring force, $F_R \propto -x$

$$\text{or } F_R = -cx$$

where c is force constant

Frictional force, $F_D \propto -\frac{dx}{dt}$

$$\text{or } F_D = -Y \frac{dx}{dt}$$

where Y is damping co-eff. of resistive medium

$$F = F_R + F_D = (-cx - Y \frac{dx}{dt})$$

A/c to Newton's 2nd law of motion

$$F = ma = m \frac{d^2x}{dt^2}$$

$$m \frac{d^2x}{dt^2} = (-cx - Y \frac{dx}{dt})$$

$$\text{or } \frac{d^2x}{dt^2} = -\frac{c}{m}x - \frac{Y}{m} \frac{dx}{dt}$$

$$\text{or } \frac{d^2x}{dt^2} + \frac{c}{m}x + \frac{Y}{m} \frac{dx}{dt} = 0$$

$$\text{or } \frac{d^2x}{dt^2} + \frac{Y}{m} \frac{dx}{dt} + \frac{c}{m}x = 0$$

Put $\frac{Y}{m} = 2k$, k is damping constant

$$\frac{c}{m} = \omega_0^2$$

$$\frac{d^2x}{dt^2} + 2k \frac{dx}{dt} + \omega_0^2 x = 0$$