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Topic: Application of Derivative

Second and Higher Ordered Derivative

Given a function y = f(x) , its derivative w.r.t x i.e. dy/dx is called the first derivative.

Since dy/dx is also a function of x it can be differentiated further. The derivative of dy/dx w.r.t x is termed as the second derivative of y denoted by d(dy/dx)/dx = d2
y/dx2 proceeding in a similar manner we can obtain the third derivative d3y/dx3,
fourth derivative d4y/dx4,...the nth derivative dny/dxn.

The first, second, third...nth derivatives are also denoted by f′(x) f″(x), f‴(x),...fn(x).

Interpretation of derivatives of various orders

The first derivative ,dy/dx, of the function y = f(x) , measures the instantaneous rate of change of y. In terms of figure, dy/dx gives the slope of the curve at a point or equivalently the slope of the tangent to the curve at that point. If dy/dx C (<0) at a point, this implies that y increases (decreases) for a small increase in the value of x.

Further if dy/dx = 0 it implies that y does not change. (or remains stationary) for a small increase in the value of x.

The second derivative d2y/dx2 gives the rate of change of dy/dx. Thus if d2y/dx2, the slope of the curve is increasing and if d2y/dx2 <0, the slope of the curve is decreasing, as we pass through the concerned point.

Second derivative criterion for concavity(convexity) of a function.

The sign of the second derivative is related to the curvature of the function y = f(x).

When d2y/dx2 0 over a domain or its subset the curve is strictly convex from below and when d2y/dx2 <0, the curve is strictly concave from below. If d2y/dx2or 0 the function is said to be weakly convex (concave). It may be pointed out that the condition d2y/dx2(<)0 is not necessary but sufficient for convexity (concavity).

Further a negative value d2y/dx2 can be associated with negative,positive or zero value of dy/dx.

Such a situation is represented by the curve of figure(i), where d2y/dx2 <0 for all values of x.

Similarly a positive value of d2y/dx2 may also be associated with a negative,positive or zero value of dy/dx.

Cost Functions

The cost of production (C) expressed as a function of the level of output (x) is termed as cost function written as TC = F(x). The short run cost function can be derived from the short run production function. In short run the quantities of certain factors remain fixed while the quantities of others are variable. The cost of using fixed factors is known as fixed cost and the cost of using variable factors is known as variable cost. Fixed cost does not vary with the level of output i.e. it remains constant whether a firm produces zero or x (>0) units of output. On the other hand variable cost increases as more and more units of output are produced.

Further variable cost is zero when x = 0 i.e. the firm is not producing anything. We can write,

Total Cost (TC) = Total Fixed Cost (TFC) + Total Variable Cost (TVC)

The nature of variable cost depends upon the nature of short run production function.

When, as a result of increment in the variable factor, the output increases at increasing rate, the variable cost will increase at decreasing rate i.e. d(TVC)/dx > 0 and d2(TVC)/dx2<0 .

When, as a result of increment in the variable factor, the output increases at decreasing , the variable cost will increase at increasing rate i.e. d(TVC)/dx >0 and d

2(TVC)/dx2 < 0.

This curve has two turning points and so can be represented by a cubic polynomial of the type TVC = ax3 + bx2 + cx where, a, b, c are constants with a,c >0, b<0 and b2 > 3ac and x denotes the level of output and TFC is denoted by d. Thus can write total cost function

TC = ax3 + bx2 + cx + d

The Average Cost, AC is given by

AC = TC/x = TFC/x + TVC/x = AFC + AVC

where AFC is TFC/x is average fixed cost and AVC = TVC/x is average variable

cost.

Under the normal conditions the average variable cost is AVC = (ax3- bx2 + cx)/x = ax2 + bx + c (a,b,c > 0 and b2 < 3ac), which is the equation of a parabola with axis pointing vertically upward.

Further, the Marginal Cost, MC is given by

MC = d(TC)/dx = d(TVC)/dx = 3ax2- 2bx + c, which is also the equation of parabola with axis pointing vertically upwards.

The shape of the marginal cost function depends on the total cost function since the marginal cost is the slope of the total cost. Initially when the total cost is increasing at a decreasing rate, the marginal cost is decreasing. When the total cost starts increasing at an increasing rate, the marginal cost also starts increasing.