3.11.14. Critical Damping Resistance (CDR)

We know that the electromagnetic damping is based on the eddy current generation currents within a conductive material in the presence of a magnetic field. A damping force that resists the system's motion is produced when these eddy currents develop their own magnetic fields that counter the initial magnetic field (Lenz's law). Critical Damping Resistance is defined as the ratio of the damping force to the critical damping force of the system. The CDR evaluates how well the electromagnetic dampening system dissipates energy and slows down the motion of the object.

Thus, CDR = Damping force/Critical damping force

The minimum amount of damping needed to completely stop the system in the shortest possible time without any oscillation or overshoot is represented by the critical damping force. It is the ideal damping situation. The stronger damping is denoted by a higher CDR, which implies that the electromagnetic damping system is more effective in reducing the motion of the object. This means that the damping force produced by the eddy currents is equal to or greater than the critical damping force, leading to a quicker energy dissipation and faster dissipation of energy of the system's oscillations or vibrations.

Example 51: Determine the deflection when a steady current of 3μA is passed through which a ballistic galvanometer gives a throw of 10cm when a charge of 1.2×10^{-6} coulomb is passed. Its time period is 3.14sec.

Solution: Given that, $d_q = 10cm$, $q = 1.2\times10^{-6}$ coulomb
$d_i = ?$, $I = 3\times10^{-6}$ amp. and $T = 3.14sec$.

The charge sensitivity and current sensitivity relation, $Qs = \frac{2\pi}{T}C_s$

where $Qs = \frac{d_q}{q}$ and $C_s = \frac{d_i}{i}$; So, $\frac{10}{1.2\times10^{-6}} = \frac{2\times3.14}{3.14}\times\frac{d_i}{3\times10^{-6}}$

Thus, $\frac{10}{1.2\times10^{-6}} = \frac{2\times3.14}{3.14}\times\frac{d_i}{3\times10^{-6}}$; Or d = 12.5cm

Example 52: Find the ballistic constant of the galvanometer when a condenser of 1,000 pf is charged to a potential difference of 1 volt and then discharged through a B.G. The first throw on a scale 1 metre away is 62.2cms. If the time period of free vibrations be 10sec and logarithmic decrement 0.02.

Solution: As we know that,

$q = CV = 1000\times10^{-12}\times1 = 1\times10^{-9}$ coul. and also, $q = K\theta_1\left(1+\frac{\lambda}{2}\right)$

Where $K = \frac{T}{2\pi}\frac{c}{NBA}$, is known as galvanometer constant

Here, $\theta_1 = 62.2cm$, $q = 1\times10^{-9}$ coul, $\lambda = 0.02$

Thus, $1\times10^{-9} = K\times62.2\times10^{-2}\left(1+\frac{0.02}{2}\right)$ or $K = \frac{1\times10^{-9}}{62.2\times10^{-2}\times1.01}$ coulomb/metre

Example 53: The first three successive deflections of a ballistic galvanometer are 15, 14.9 and 14.8cm. Determine the first corrected deflection under damping.

Solution: Given that, $\theta_1 = 15\text{cm}$, $\theta_2 = 14.9\text{cm}$ and $\theta_3 = 14.8\text{cm}$

First corrected deflection is given by:

$$\theta = \theta_1\left(1+\frac{\lambda}{2}\right) \quad \ldots(1)$$

Logarithmic decrement is given by: $\lambda = \frac{2.303}{(n-1)}\log_{10}\left(\frac{\theta_1}{\theta_n}\right)$

First three successive defelctions $\Rightarrow n = 3$

$$\Rightarrow \quad \lambda = \frac{2.303}{(3-1)}\log_{10}\left(\frac{\theta_1}{\theta_3}\right) \quad \Rightarrow \quad \lambda = \frac{2.303}{2}\log_{10}\left(\frac{15\times10^{-2}}{14.8\times10^{-2}}\right)$$

$$\Rightarrow \quad \lambda = 6.71\times10^{-3} \quad \ldots(2)$$

Put the values of equation (2) in equation (1), we get

$$\theta = 15\times10^{-2}\left(1+\frac{6.71\times10^{-3}}{2}\right) \Rightarrow \theta = 15.05\text{cm}$$

Hence, the first corrected deflection in the absence of damping is 15.05cm.

Example 54: The successive deflections to the right and left of the mean position in the case of a ballistic galvanometer are 25.0, 24.9 and 24.8cm. Find the deflection without damping.

Solution: We have $\theta_1 = 25.0\text{cm}$, $\theta_2 = 24.9$ cm and $\theta_3 = 24.8\text{cm}$

The logarithmic decrement λ is calculated as,

$$\lambda = \frac{1}{2}\log_e\frac{\theta_1}{\theta_3} = \frac{1}{2}\times2.3026\times\log_{10}\frac{\theta_1}{\theta_3}$$

$$\log_{10}\frac{\theta_1}{\theta_3} = \log_{10}25.0 - \log_{10}24.8 = 1.3979 - 1.3945 = 0.0034$$

$$\therefore \quad \lambda = \frac{1}{2}\times2.3026\times0.0034 = 0.0039$$

The deflection without damping is calculated as:

$$\theta_0 = \theta_1\left(1+\frac{\lambda}{2}\right) = 25.0\left(1+\frac{0.0039}{2}\right) = 25.05\text{cm}$$

Example 55: A capacitor charged to 5V is discharged through a ballistic galvanometer and it shows a corrected deflection of 7.5cm. The current sensitivity is 5mm/μA and the period of oscillation of the ballistic galvanometer is 10s. Determine the capacitance of the capacitor.

Solution: Given that, $V = 5\text{V}$, $\theta = 7.5\text{cm} = 7.5\times10^{-2}\text{m}$, $T = 10\text{s}$ and

$$\frac{\theta}{i} 5\,\text{mm}/\mu\text{A} = 5\times10^3\,\text{m}/\text{A}$$

Charge stored in the capacitor is given by:

$Q = \frac{C}{NBA}\left(\frac{T}{2\pi}\right)\theta$ and $\frac{\theta}{i} = \frac{NBA}{C}$

$\Rightarrow Q = \left(\frac{i}{\theta}\right)\left(\frac{T}{2\pi}\right)\theta N \quad \Rightarrow Q = \left(\frac{1}{5\times10^{3}}\right)\times\left(\frac{10}{2\pi}\right)\times7.5\times10^{-2}$

$\Rightarrow Q = 2.38 \times 10^{-5}$ C

Capacitance of the capacitor is given by:

$$C' = \frac{Q}{V} \Rightarrow C' = \frac{2.38\times10^{-5}}{5} = 4.76\mu F$$

Hence, the capacitance of the capacitor is $4.76\mu F$.

Example 56: The current sensitivity of a ballistic galvanometer is 3.5×10^{-4} mm/A. If the period of the coil is 5s then find the charge sensitivity of the galvanometer.

Solution: Given that, current sensitivity = 3.5×10^{-4}mm/A = 3.5×10^{-7}m/A and T = 5s

$$\text{Charge sensitivity} = \left(\frac{2\pi}{T}\right)\text{Current sensitivity}$$

$$\Rightarrow \text{Charge sensitivity} = \left(\frac{2\pi}{5}\right)\times3.5\times10^{-7}$$

$$\Rightarrow \text{Charge sensitivity} = 4.39 \times 10^{-7}\text{m/C}$$

Hence, the charge sensitivity of a ballistic galvanometer is 4.39×10^{-7}m/C.

3.12. MULTIPLE CHOICE QUESTIONS

1) To which law of electricity is Biot-Savart law of magnetism analogous to
   a) Coulomb's law b) Ohm's law
   c) Kirchoff's law d) Faraday's law

2) What is the magnetic field outside a solenoid?
   a) Infinity b) Half the value of the field inside
   c) Double the value of the field inside d) Zero

3) Which, among the following qualities, is not affected by the magnetic field?
   a) Moving charge b) Stationary charge
   c) Change in magnetic flux d) Current flowing in a conductor

4) What is the strength of magnetic field known as ________.
   a) Flux b) Density
   c) Magnetic flux density d) Magnetic strength

5) When a straight conductor is carrying current:
   a) There are circular magnetic field lines around it
   b) There are magnetic field lines parallel to the conductor
   c) There are no magnetic field lines
   d) None of the above