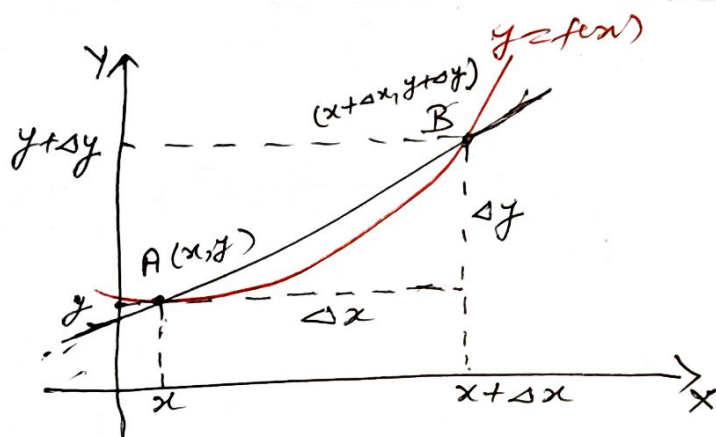


Definition of derivative:—

Let  $y = f(x)$  is a function of single variable  $x$ .

Draw a graph of  $f(x)$  in Cartesian  $(x, y)$  coordinates



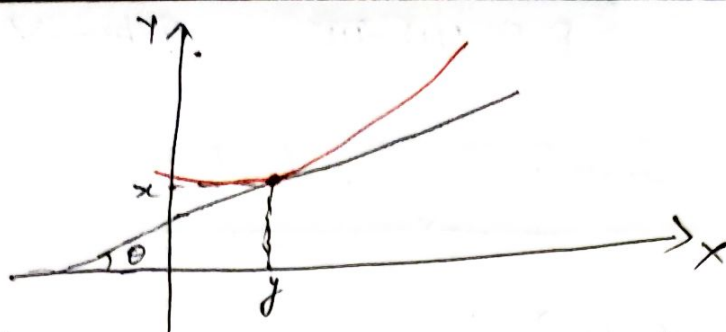
⊙ Consider two points A and B on  $y = f(x)$ .

Coordinates of A and B are  $(x, y)$  and  $(x + \Delta x, y + \Delta y)$  respectively

If we write down the slope of line joining A & B, it is given as

$$\tan \theta = \frac{\Delta y}{\Delta x} = \frac{(y + \Delta y) - y}{\Delta x}$$

If Point A moves along the curve towards B, then  $\Delta y$  and  $\Delta x \rightarrow 0$ , ~~but~~ but ratio  $\frac{\Delta y}{\Delta x}$  need not to necessarily vanish.



when  $\Delta y \rightarrow 0$   
 $\Delta x \rightarrow 0$ , line AB becomes tangent to the curve at Point A

$\Rightarrow \tan \theta \rightarrow$  slope of tangent at A ( $=m$ )

$$\text{or } m = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{(y + \Delta y) - y}{\Delta x}$$

$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \rightarrow$  derivative of  $y$  with respect to  $x$

$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$  is written as  $\frac{dy}{dx}$ .

Now for  $y = f(x)$  and  $y + \Delta y = f(x + \Delta x)$

$$\frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \left[ \frac{f(x + \Delta x) - f(x)}{\Delta x} \right]$$

H.W. (i) Obtain the derivatives of the trigonometric functions:

$\sin x, \cos x, \tan x, \cot x, \operatorname{cosec} x, \operatorname{sec} x$ .

(ii) Hyperbolic functions  $\sinh x, \cosh x, \tanh x, \operatorname{sech} x, \operatorname{cosech} x, \coth x$

(iii) Plot the graph of above functions.

(iv) Draw the graph of  $y = |x| + |x-1|$

(v) Examine the differentiability of the function

$$f(x) = x^m \sin\left(\frac{1}{x}\right) \text{ when } x \neq 0, m > 0$$

$$f(x) = 0, \text{ when } x = 0,$$

at point  $x = 0$ . Determine  $m$  when  $f'(x)$  is continuous at the origin.