10 → Stat and explain Fourier's heat flow equation.

⇒ Fourier's heat flow → If ~~a~~ a solid of volume v be enclosed by a surface S, the temperature of which is not the same at all ~~pp~~ points. Heat would thus flow from places at higher temperature to those at lower by conduction. The amount of heat flowing out of a small surface ds in time Δt is

$$dQ = -\left[(K\vec{\nabla}\theta)\cdot\hat{n}\,ds\right]\Delta t$$

Where K is thermal conductivity, θ the temperature at (x, y, z, t) and $\hat{n}$ the outward unit ~~~~ S (is normal)

∴ Total amount of heat flowing out of the whole surface S in time Δt is

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$$Q = -\Delta t \iint_S K \vec{\nabla}\theta \cdot \hat{n}\, ds$$

$$= -\Delta t \iiint_V \vec{\nabla} \cdot (K \vec{\nabla}\theta)\, dv \quad \text{——— (i)}$$

By the divergence theorem.

If c be the specific heat of the material, ρ its density and $\partial\theta/\partial t$ the rate of increase of temperature within the volume V of the body, the amount of heat flowing in time Δt is (out of V)

$$Q = -\Delta t \iiint_V c\rho \frac{\partial \theta}{\partial t}\, dv \quad \text{——— (ii)}$$

from (i) and (ii), therefore

$$\iiint_V \left\{ c\rho \frac{\partial \theta}{dt} - \vec{\nabla} \cdot (K \vec{\nabla}\theta) \right\} dv = 0$$

Since dv is arbitrary, $c\rho \frac{\partial \theta}{\partial t} = \vec{\nabla} \cdot (K \vec{\nabla}\theta)$

or, $$\boxed{\frac{\partial \theta}{\partial t} = \frac{K}{\rho c} \nabla^2 \theta = h \nabla^2 \theta} \quad \text{——— (iii)}$$

Where $h = K/\rho c$ it's the thermal diffusivity of material

eqn (iii) is the three-dimensional fourier's heat flow eqn

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