Interference and Diffraction

14.1 Superposition of Waves

Consider a region in space where two or more waves pass through at the same time. According to the superposition principle, the net displacement is simply given by the vector or the algebraic sum of the individual displacements. Interference is the combination of two or more waves to form a composite wave, based on such principle. The idea of the superposition principle is illustrated in Figure 14.1.1.

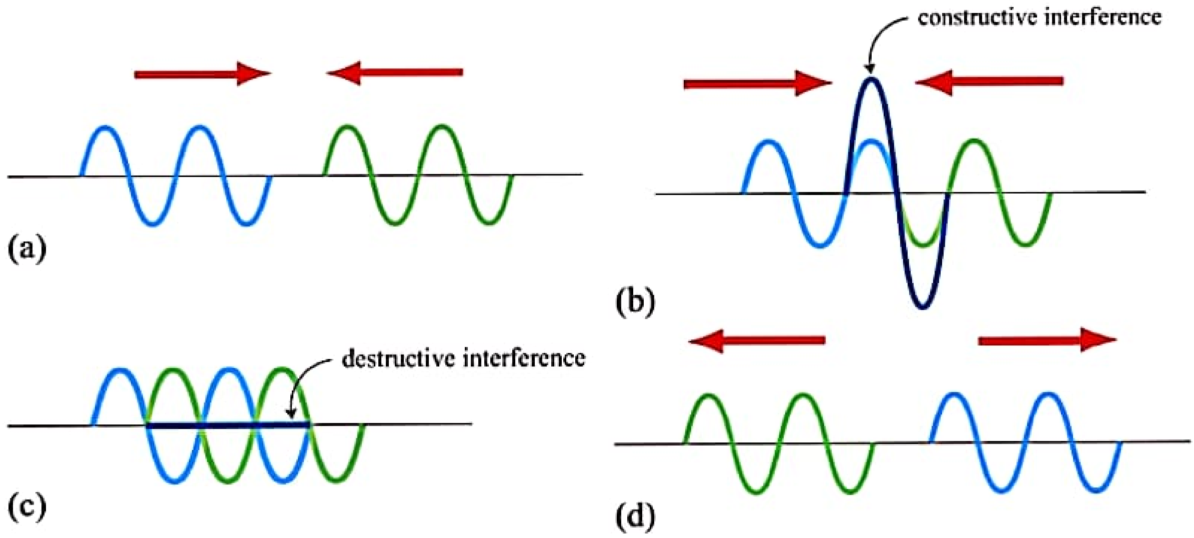

Figure 14.1.1 Superposition of waves. (b) Constructive interference, and (c) destructive interference.

Suppose we are given two waves,

$$\psi_1(x,t) = \psi_{10}\sin(k_1 x \pm \omega_1 t + \phi_1), \qquad \psi_2(x,t) = \psi_{20}\sin(k_2 x \pm \omega_2 t + \phi_2) \tag{14.1.1}$$

the resulting wave is simply

$$\psi(x,t) = \psi_{10}\sin(k_1 x \pm \omega_1 t + \phi_1) + \psi_{20}\sin(k_2 x \pm \omega_2 t + \phi_2) \tag{14.1.2}$$

The interference is constructive if the amplitude of $\psi(x,t)$ is greater than the individual ones (Figure 14.1.1b), and destructive if smaller (Figure 14.1.1c).

As an example, consider the superposition of the following two waves at $t = 0$:

$$\psi_1(x) = \sin x, \qquad \psi_2(x) = 2\sin\left(x + \frac{\pi}{4}\right) \tag{14.1.3}$$

The resultant wave is given by

$$\psi(x) = \psi_1(x) + \psi_2(x) = \sin x + 2\sin\left(x + \frac{\pi}{4}\right) = \left(1+\sqrt{2}\right)\sin x + \sqrt{2}\cos x \qquad (14.1.4)$$

where we have used

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad (14.1.5)$$

and $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$. Further use of the identity

$$\begin{aligned} a\sin x + b\cos x &= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right] \\ &= \sqrt{a^2+b^2}\left[\cos\phi\,\sin x + \sin\phi\,\cos x\right] \\ &= \sqrt{a^2+b^2}\,\sin(x+\phi) \end{aligned} \qquad (14.1.6)$$

with

$$\phi = \tan^{-1}\left(\frac{b}{a}\right) \qquad (14.1.7)$$

then leads to

$$\psi(x) = \sqrt{5+2\sqrt{2}}\,\sin(x+\phi) \qquad (14.1.8)$$

where $\phi = \tan^{-1}(\sqrt{2}/(1+\sqrt{2})) = 30.4° = 0.53$ rad. The superposition of the waves is depicted in Figure 14.1.2.

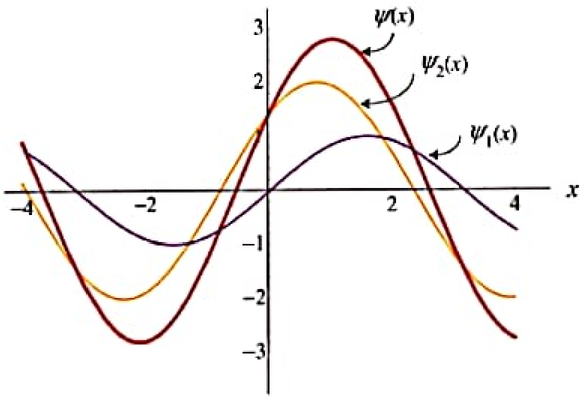

Figure 14.1.2 Superposition of two sinusoidal waves.

We see that the wave has a maximum amplitude when $\sin(x+\phi) = 1$, or $x = \pi/2 - \phi$. The interference there is constructive. On the other hand, destructive interference occurs at $x = \pi - \phi = 2.61$ rad, where $\sin(\pi) = 0$.