Carnot's cycle (part II)

Work done by the engine per cycle

During the above cycle, the working substance absorbs an amount of heat Q_1 from the source and rejects Q_2 to the sink.

Hence, the net amount of heat absorbed by the gas per cycle $= Q_1 - Q_2$.

The net work done by the engine per cycle

$$= W_1 + W_2 + W_3 + W_4$$

$$= W_1 + W_3 \qquad (\because W_2 = -W_4)$$

from the graph, the net work done per cycle

= area ABGEA + area BCHGB − area CHFDC − area DFEAD.

= area ABCDA ——— (5)

Thus, the area enclosed by the carnot cycle consisting of two isothermals and two adiabatics gives the net amount of work done per cycle.

In cyclic process, Net Heat absorbed = Net Work done per cycle.

$$\Rightarrow Q_1 - Q_2 = W_1 + W_3$$

$$= RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4} \quad \text{——— (6)}$$

Since the points A & D lie on same adiabatic DA;

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \quad \text{——— (7)}$$

Similarly, points B and C lie on the same adiabatic BC.

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \quad \text{——— (8)}$$

from eqn (7) & (8)

$$\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

or, $\quad \dfrac{V_1}{V_4} = \dfrac{V_2}{V_3}$

or, $\quad \dfrac{V_2}{V_1} = \dfrac{V_3}{V_4}$ putting in eqn (6) we get

$$\text{Net work done} = Q_1 - Q_2$$
$$= RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_2}{V_1}$$

$$\Rightarrow \omega = (Q_1 - Q_2)$$
$$= R(T_1 - T_2) \log_e \frac{V_2}{V_1} \quad \text{——— (9)}$$

——— x ———