

Carnot's cycle

The process occurring in the engine are reversible. The reversible cyclic process consists of a sequence of isothermal & adiabatic curves on a $p-v$ diagram, and is known as Carnot's cycle.

The state of a system is uniquely determined by a point on a $p-v$ diagram known as indicator diagram, each point representing an equilibrium state. A line joining a series of such points represents a succession of an infinitely large number of equilibrium states infinitesimally close to each other and hence depicts a quasi-static process. A quasi static process is a process carried out sufficiently slowly so that the system under consideration remains at all times arbitrarily close to equilibrium.

Work done in Carnot's cycle: - In

Carnot's cycle, the working substance is supposed to undergo the following

four quasi-static operations as shown in fig (1)

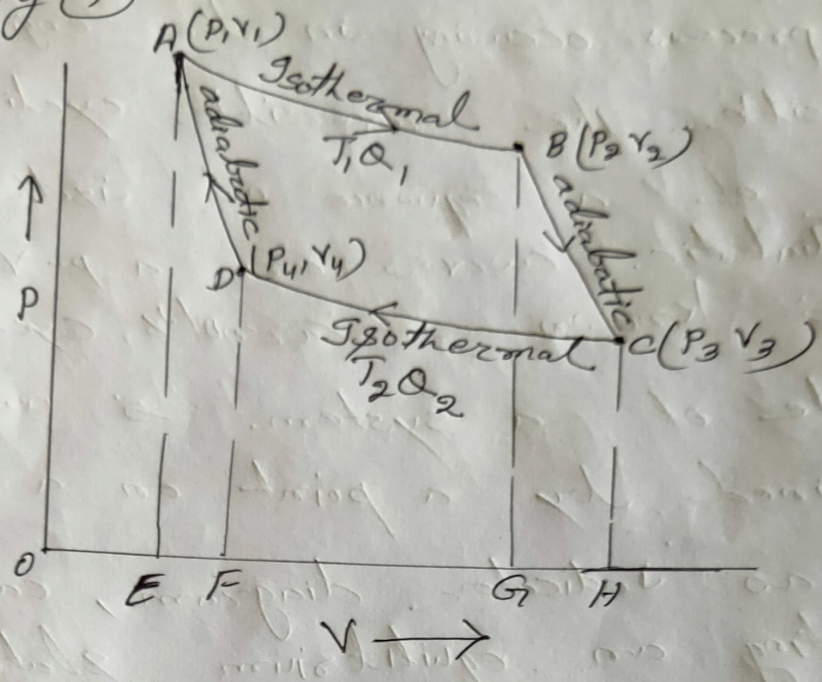


Fig (1)

1. Isother thermal expansion:- Suppose the cylinder contains one mole of perfect gas & initial temperature be T₁ K. in the cylinder and of heat source. The cylinder is first placed on the source, so that the gas acquire the temperature T₁ of the source. It is then allowed to undergo quasi-static expansion.

Since the gas expands, its temperature tends to fall. Heat passes into the cylinder through the perfectly conducting base which is in contact with the source. The gas therefore, undergoes slow isothermal expansion at the constant temperature T_1 .

Let the working substance during isothermal expansion go from its initial state $A(p_1, v_1, T_1)$ to heat Q_1 from the source at T_1 and does work W_1 given by

$$\begin{aligned} Q_1 = W_1 &= \int_{v_1}^{v_2} p \, dv \\ &= RT_1 \log_e \frac{v_2}{v_1} \\ &= \text{Area ABGEA} \quad \text{--- (1)} \end{aligned}$$

2. Adiabatic expansion: -

$$\begin{aligned} W_2 &= \int_{v_2}^{v_3} p \, dv = k \int_{v_2}^{v_3} \frac{dv}{v^\gamma} \\ &= \frac{k v_3^{1-\gamma}}{1-\gamma} - \frac{k v_2^{1-\gamma}}{1-\gamma} \end{aligned}$$

$$\Rightarrow W_2 = \frac{P_3 V_3 - P_2 V_2}{1-\gamma} = \frac{RT_2 - RT_1}{1-\gamma}$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1} = \text{Area ACHGB} \quad \text{--- (2)}$$

3. Isothermal compression: -

$$Q_2 = W_3 = \int_{V_3}^{V_4} p \, dV = RT_2 \log \frac{V_4}{V_3} \quad \text{--- (3)}$$

$$= -RT_2 \log \frac{V_3}{V_4} = \text{Area CHFDC}$$

4. Adiabatic compression: -

$$W_4 = \int_{V_4}^{V_1} p \, dV$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1}$$

$$= \text{Area DFEAD} \quad \text{--- (4)}$$