eg :- $\{1, 2, 3\}$.

(L-4).

Def: Unbounded above set :- A non-empty subset S of $\mathbb{R}$ is said to be unbounded above if it is not bounded above.
i.e. for each real number K we can find at least one element $x \in S$ such that
$x > K$, $x \in S$

Example: (1) if $S = \{1, 2, 3, 4\}$, then S is bounded above by 4 and any number greater than 4 is also upper bound of S.

(2) The set of negative integers i.e. $\{\cdots, -3, -2, -1\}$ is a bounded above set.

(3) The set $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, I, $\mathbb{R}$ all are unbdd above sets.

(4) The intervals (a, b), $[a, b]$, $(-\infty, a)$, $(-\infty, b]$ all are bounded above sets whereas (a, ∞), $[a, \infty)$, $(-\infty, \infty)$ are unbdd above.

Bounded below set :- A non-empty subset S of $\mathbb{R}$ is said to be bounded below if there exists a real number K s.t. $K \leq x \ \forall \ x \in S$.

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This real number K is said to be a lower bound of S. If S is bounded below then it has infinite number of lower bounds

e.g.:- $\{1, 2, 3\}$

→ **Unbounded below set :-** A non-empty subset S of R is s.t.b unbounded below if it is not bounded below.
i.e, for each real number K we can find at least one element $x \in S$ s.t $x < K$

Example (1) If $S = \{1, 2, 3, 4\}$, then S is bounded below by 1 & any number less than 1 is a lower bound of S. Infact every finite set is bounded below

(2) The set of positive integer is bounded below.

(3) The sets Z, Q, I, R all are unbounded below sets

(4) The intervals (a, b), $[a, b]$, (a, ∞), $[a, \infty)$ all are bounded below sets.

Bounded set :- A non-empty subset S of R is said to be bounded if it is both bounded above and bounded below.
i.e, if ∃ two real numbers k and K such that $k \le x \le K$ $\forall x \in S$.

Note (1) Any finite set $A \subseteq R$ is bounded set
(2) Any infinite set $A \subseteq R$ may or may not be bounded.

Ex:- N is unbounded set while $[1, 2]$ is bounded set & both are infinite set.

Unbounded set :- A non-empty set S of R is said to be unbdd set if it is unbdd. otherways.

either it is unbdd above or unbdd below or both.

Ex :- N, Q, IR, Z etc

→ Greatest element :- let S be a non-empty subset of IR. A real number 'g' is called greatest element of S if

(1) $\boxed{g \in S \text{ and}}$ $x \leq g \quad \forall x \in S$.

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Ex :- $S = \{1, 2, 3, 4\}$, greatest element of S is 4. ($\because 4 \in S$ & $4 > 1, 2, 3$)

& $B = \{1, 2, 3, \ldots\}$ greatest elt doesn't exist.

$\boxed{C = (3, 5)}$ greatest element $5 \notin C$.

⇒ C has no greatest element.

i.e., greatest elt of C doesn't exist

→ Least element :-

Let S be a non-empty subset of IR, a real number 'l' is called least element of S if (a) $\boxed{l \in S}$

(b) $l \leq x \quad \forall x \in S$

Example :- $S = \{1, 2, 3, 4\}$ least element of S is 1

$B = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$ least element of B does not exist

$\boxed{C = (3, 4)}$ ← least element of C not exist. {∵ least (lower) elt of C is 3 & 4 ($3 \notin C$.

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* Least Upper Bound (l.u.b) :-

Let S be a non-empty subset of $\mathbb{R}$. A real number u is said to be a least upper bound (l.u.b) or supremum of S if

(1) $x \le u \ \forall x \in S$ i.e, u is upper bound of S

(2) If u' is any upper bound of S, then $u \le u'$.

Ex:- (1) If S is a finite set, then its greatest element works as l.u.b.

For example:- If $S = \{1, 2, 3, 4\}$ then 4 is the l.u.b of S

(2) If $S = (2, 3)$ then 3 is l.u.b

(3) If $S = [1, 7]$ then 7 is l.u.b.

Remark:- It is clear that the l.u.b of set may or may not belong to the set

* Greatest Lower bound (g.l.b) :-

Let S be a non-empty subset of $\mathbb{R}$. A real number l is said to be greatest lower bound (g.l.b) or infimum of S if

(1) $l \le x \ \forall x \in S$ i.e, l is lower bound of S

(2) If l' is any lower bound of S then $l' \le l$.

Result (1) Supremum of a set may or may not exist

(2) least upper bound (or supremum) of a set if exists, it is unique