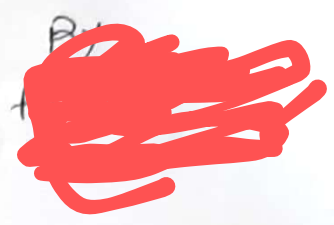


Infinite series



\* Comparison test

If  $\sum U_n$  and  $\sum V_n$  be two series of (+)ve terms then

If  $\lim_{n \rightarrow \infty} \frac{U_n}{V_n}$  be finite and non-zero, the series will be both convergent or both convergent.

\* The infinite series

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots \text{ to } \infty \text{ is}$$

convergent if  $p > 1$  and  
divergent if  $p \leq 1$ .

Working rule

1. Find the  $n^{\text{th}}$  term  $U_n$ .

2. consider another auxiliary series whose  $n^{\text{th}}$  term ( $V_n$ ) is equal to

$$V_n = \frac{\text{term of highest power of } n \text{ in numerator of } U_n}{\text{term of highest power of } n \text{ in the denominator of } U_n}$$

3. Find  $\frac{U_n}{V_n}$ .

4. Find  $\lim_{n \rightarrow \infty} \frac{U_n}{V_n}$ .

If  $\lim_{n \rightarrow \infty} \frac{U_n}{V_n} =$  finite and non-zero

then proceed as following.

5. compare  $V_n$  with the series  $\sum \frac{1}{n^p}$ .

$\sum \frac{1}{n^p}$  ~~which~~ is cgt if  $p > 1$ ,

is dgt. if  $p \leq 1$ .

6. By comparison test both the series  $\sum U_n$  and  $\sum V_n$  will converge or diverge simultaneously.

## EXAMPLES

1. Test the convergence of the series

$$1 + \frac{1+2}{1+2^2} + \frac{1+3}{1+3^2} + \dots + \frac{1+n}{1+n^2} + \dots$$

Soln The series can be written as

$$\frac{1+1}{1+1^2} + \frac{1+2}{1+2^2} + \frac{1+3}{1+3^2} + \dots + \frac{1+n}{1+n^2} + \dots$$

Then its  $n^{\text{th}}$  term ( $=U_n$ ) =  $\frac{1+n}{1+n^2}$

Let us consider another series whose  $n^{\text{th}}$  term is  $V_n$  and  $V_n = \frac{n}{n^2} = \frac{1}{n}$

$$\begin{aligned} \therefore \frac{U_n}{V_n} &= \frac{\frac{1+n}{1+n^2}}{\frac{1}{n}} = \frac{n(1+n)}{1+n^2} \\ &= \frac{n+n^2}{1+n^2} = \frac{n^2\left(\frac{1}{n}+1\right)}{n^2\left(\frac{1}{n^2}+1\right)} \\ &= \frac{1+\frac{1}{n}}{1+\frac{1}{n^2}} \end{aligned}$$

$$\therefore \lim_{n \rightarrow \infty} \frac{U_n}{V_n} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1+\frac{1}{n^2}} = 1$$

which is finite and non-zero.

So, by comparison test, the two series  $\sum U_n$  and  $\sum V_n$  behave alike (i.e. converge or diverge simultaneously).

Now  $V_n = \frac{1}{n} \Rightarrow \sum V_n = \sum \frac{1}{n}$

$\Rightarrow \sum V_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots \rightarrow \infty$

Comparing it with  $\sum \frac{1}{n^p}$ , we have  $p=1$ .

$\Rightarrow \sum V_n$  is not convergent, i.e. divergent.

Hence, by comparison test,  $\sum U_n$  is div.

Q) Test the convergency of the series

$$\sum \frac{5n-3}{2n^3-1}$$

Soln Here,  $U_n = \frac{5n-3}{2n^3-1}$

$\Rightarrow$  ~~the~~ consider another series  $\sum V_n$  with

$$V_n = \frac{n}{n^3} = \frac{1}{n^2}$$

$$\therefore \frac{U_n}{V_n} = \frac{n^2(5n-3)}{2n^3-1} = \frac{\frac{n^2(5n-3)}{n^3}}{\frac{2n^3-1}{n^3}} = \frac{5 - \frac{3}{n}}{2 - \frac{1}{n^3}}$$

$$\therefore \lim_{n \rightarrow \infty} \frac{U_n}{V_n} = \lim_{n \rightarrow \infty} \left( \frac{5 - \frac{3}{n}}{2 - \frac{1}{n^3}} \right) = \frac{5}{2}$$

which is finite and non-zero

So, by comparison test, they behave alike.

But  $\sum V_n = \sum \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \dots + \frac{1}{n^2} + \dots$  is convergent

$\Rightarrow \sum U_n$  (the given series) is also convergent.