* **Completeness Axioms**:- Every non-empty bdd above set of real number has a least upper bound in $\mathbb{R}$

Complete set:- A subset S of real number is said to be complete if every bdd above subset of S has the least upper bound in S.

→ In the view of above definition, the Completeness Axioms can be stated in the following manner:

Result: (1) the set $\mathbb{R}$ of real number is complete.
(2) $\mathbb{Q}$ is not complete,
∴ Let $A = \{x : 2 < x^2 < 3 ; x \in \mathbb{Q}\}$

$= \{x : x \in \mathbb{Q}, \sqrt{2} < x < \sqrt{3}\}$

$\subseteq \mathbb{Q}$ & $\sqrt{3} \notin \mathbb{Q}$.

is not complete

(3) Every non-empty bounded below subset of real number has the greatest lower bound. (infimum)

** **Archimedian properties of $\mathbb{R}$**: If a and b are two positive real numbers, then $\exists$ a natural number n s.t. $na > b$.

$a>0, b>0, \exists n \in \mathbb{N}$

Corollary (1): If $a > 0$, then $\exists$ a natural number n s.t. $n > a$.

$a>0$, b/a, $n > b/a$, $na > b$

Corollary (2): for $\epsilon > 0$ however small, then $\exists$ a $n \in \mathbb{N}$ s.t. $\frac{1}{n} < \epsilon$

$\epsilon > 0$, $\exists n \in \mathbb{N}$, $n > \frac{1}{\epsilon}$

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Corol(3):- for any real number x $\exists$ two integer m and n such that $m < x < n$.

Corollary(4):- If n is +ve integer, $\bigcap_{n\geq 1}^{\infty}\left(1, \frac{1}{n}\right) = \phi$ $\{1\} \cap (1, \frac{1}{2}) \ldots$ $(1,0)$

Corrol(5) : If n is +ve integer, then
$$\bigcap_{n\geq 1}^{\infty}\left[0, \frac{1}{n}\right] = \{0\} = [0,1] \cap [0, \tfrac{1}{2}] \cap \cdots \cap \{0\}$$

Result(1) for any real number x, there exists a unique n s.t. $n \leq x < n+1$

(2) Between two distinct real numbers there are infinitely many rational numbers.

(3) Between two distinct real number there are infinitely many irrational numbers.

(4) If $A \subseteq B$ and B is bounded, then A is bounded.
Further $g.l.b(B) \leq g.l.b(A) \leq l.u.b(A) \leq l.u.b(B)$.
$A \subset B \Rightarrow \inf(B) \leq \inf(A)$ $\quad \inf A \leq \sup(A)$
$\sup A \leq \sup(B)$

(5) Let A and B are two non-empty bounded subset of $\mathbb{R}$. If $A+B = \{x+y : x \in A, y \in B\}$ then

(a) $A+B$ is bounded (b) $l.u.b(A+B) = l.u.b(A) + l.u.b(B)$
$\sup(A+B) = \sup(A) + \sup(B)$

(c) $g.l.b(A+B) = g.l.b(A) + g.l.b(B)$.
$\inf(A+B) = \inf(A) + \inf(B)$

(6) If A is any non-empty subset of $\mathbb{R}$, then
(i) $l.u.b(-A) = -g.l.b(A)$ (ii) $g.l.b(-A) = -l.u.b(A)$.

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